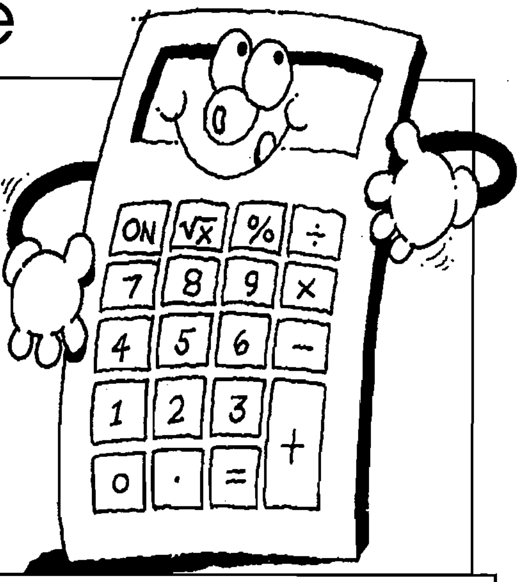
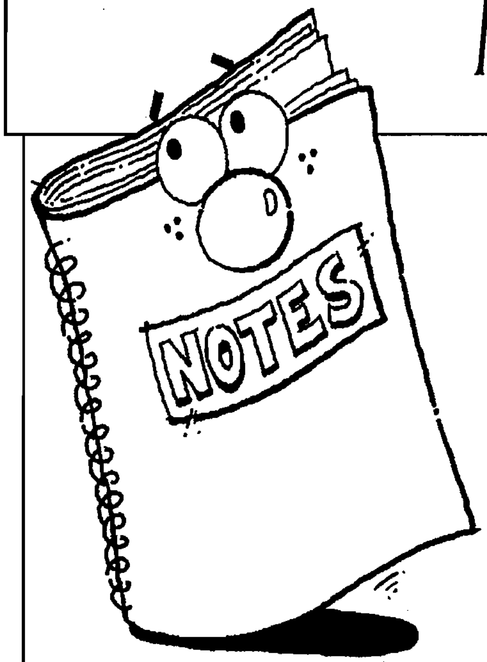
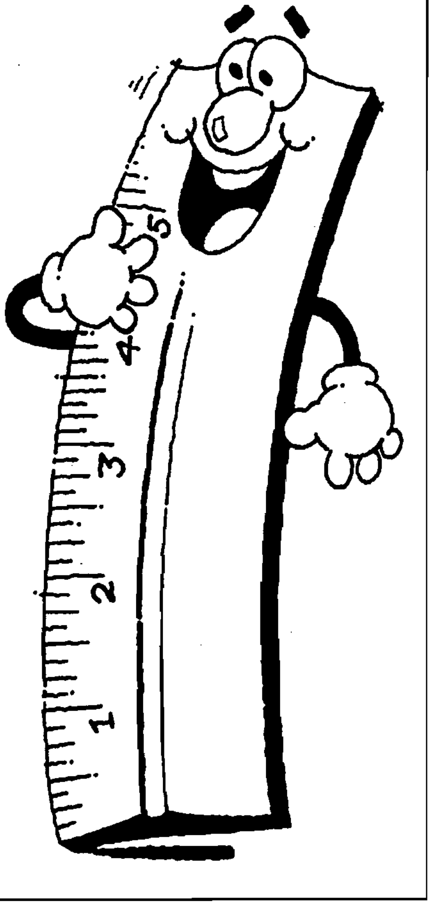
McHenry County College



MATHEMATICS REVIEW MANUAL





Basic Mathematics
Beginning Algebra
Intermediate Algebra

This manual is designed as a brief review of basic arithmetic and algebra skills. It includes a review of some important concepts, as well as sample worksheets and pretests in basic mathematics, elementary algebra, and intermediate algebra. This material **DOES NOT** reflect the content of the placement test; it is **ONLY** a review.

The mathematics placement test is a diagnostic tool to assist the college in determining the appropriate placement for you to be successful in the coursework you will be taking, not only at McHenry County College, but at any other schools you may choose to attend in the future.

Thorough understanding of the review materials will help prepare you for the placement test. However, this **DOES NOT** guarantee a passing score on the test. This material should not serve as a teaching tool, but rather as a brief review of skills covered in courses already taken. Calculators will be provided by the Testing Center.

In addition to this manual, you may wish to review arithmetic and algebra textbooks, as well as video tapes, all of which are available in the library. It is essential that you prepare for the placement test in a serious manner. The test may be taken **TWICE.** The results of the placement test will determine your mathematics placement at McHenry County College.

Math Placement Test Review Program
BOOKLET TABLE OF CONTENTS

MAT 090

Essentials of Math

ARITHMETIC PRE-TEST

1. Write $3\frac{5}{8}$ as an improper fraction

2. Write $\frac{56}{84}$ in lowest terms.

3. Multiply: $\frac{6}{7} \cdot \frac{2}{9}$

4. Divide: $\frac{3}{5} \div \frac{8}{10}$

5. Add: $\frac{3}{16} + \frac{1}{3}$

6. Subtract: $\frac{5}{9} - \frac{1}{6}$

7. Simplify: $\frac{2}{3} + \frac{5}{8} \cdot \frac{4}{3}$

8. A rare chemical is made at the rate of $1\frac{1}{4}$ grams per day. How many grams can be made in $8\frac{1}{2}$ days?

9. Write < or > to make a true statement. $\frac{5}{8}$ _____ .6

10. Solve: $\frac{x}{12} = \frac{5}{8}$

11. Write $\frac{7}{8}$ as a percent.

12. Write .06 as a fraction in lowest terms.

13. Find 42% of 830

14. Evaluate: $7 - (-9) + (-12)$

15. Evaluate: $3 \cdot 4^2 - (5 \cdot (9 - 2)) - 6^2$

ANSWERS TO ARITHMETIC PRE-TEST

1. $\frac{29}{8}$

2. $\frac{2}{3}$

3. $\frac{4}{21}$

4. $\frac{3}{4}$

5. $\frac{25}{48}$

6. $\frac{7}{18}$

7. $\frac{3}{2}$ or $1\frac{1}{2}$

8. $\frac{85}{8}$ or $10\frac{5}{8}$

9. >

10. $\frac{15}{2}$ or $7\frac{1}{2}$

11. 87.5%

12. $\frac{3}{50}$

13. 348.6 or $\frac{1743}{5}$ or $348\frac{3}{5}$

14. 4

15. −23

ARITHMETIC REVIEW: FRACTIONS

Reducing fractions:

1. Factor the numerator and denominator into prime factors.
2. Use the fact that $\frac{k}{k} = 1$ and divide out common factors.

EXAMPLE: $\frac{14}{21}$

1. $\frac{7 \text{ x } 2}{7 \text{ x } 3}$ factor numerator and denominator

2. $\frac{7}{7} \text{ x } \frac{2}{3} = \frac{2}{3}$ $\frac{7}{7} = 1$ cancel and reduce

Write a mixed number as an improper fraction:

1. <u>Multiply</u> the denominator of the fraction and the whole number.

2. <u>Add</u> to this product the numerator of the fraction.

3. Write the result of step two as the <u>numerator</u> and the original denominator as the <u>denominator.</u>

EXAMPLE: $2\frac{3}{4}$

1. $2 \text{ x } 4 = 8$ Multiply denominator and whole number

2. $8 + 3 = 11$ Add product to numerator

3. $\frac{11}{4}$ Rewrite fraction

Multiplication of Fractions:

Multiply two fractions by (1) multiplying the numerators, (2) multiplying the denominators, and (3) use cancellation, if necessary, to write the product in lowest terms.

EXAMPLE: $\frac{2}{3}\cdot\frac{5}{7}=\frac{10}{21}$ multiply numerators and denominators

EXAMPLE: $\frac{15}{28}\cdot\frac{7}{3}=\frac{\overset{5}{\cancel{15}}}{\underset{4}{\cancel{28}}}\cdot\frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{3}}}=\frac{5}{4}$ divide out common factors and multiply

Dividing Fractions:

Divide two fractions by (1) inverting the second fraction (divisor) and (2) change the division sign to multiplication.

EXAMPLE: $\frac{9}{10}\div\frac{3}{4}$

1) $\frac{9}{10}\cdot\frac{4}{3}$ invert division and change to multiplication

2) $\frac{\overset{3}{\cancel{9}}}{\underset{5}{\cancel{10}}}\cdot\frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{3}}}=\frac{6}{5}$ reduce and multiply

FRACTION WORKSHEET: MULTIPLICATION AND DIVISION

		Answers
1.	$\frac{2}{9}\cdot\frac{5}{8}$	$\frac{5}{36}$
2.	$\frac{3}{8}\cdot\frac{4}{5}\cdot\frac{2}{9}$	$\frac{1}{15}$
3.	$\frac{16}{25}\cdot\frac{35}{32}\cdot\frac{15}{64}$	$\frac{21}{128}$
4.	$\frac{7}{9}\div\frac{12}{5}$	$\frac{35}{108}$
5.	$\frac{\frac{11}{5}}{3}$	$\frac{11}{15}$
6.	$\frac{13}{40}\div\frac{26}{35}$	$\frac{7}{16}$
7.	$\frac{29}{50}\div\frac{31}{10}$	$\frac{29}{155}$
8.	$\frac{3}{4}\cdot 500$	375
9.	$\frac{2}{3}\cdot 3\frac{2}{3}\cdot\frac{6}{11}$	$\frac{4}{3}$
10.	$\frac{\frac{3}{4}}{\frac{1}{3}}$	$\frac{9}{4}$

ARITHMETIC REVIEW: FRACTIONS

Adding (or subtracting) unlike fractions:

1. Find the lowest common denominator.
2. Rewrite fractions with the common denominator
3. Add (or subtract) numerators, placing the answer over the common denominator.

EXAMPLE: $\frac{5}{12}+\frac{2}{9}$

1. The lowest common denominator is 36.

$3 \times 12 = 36$ and $9 \times 4 = 36$

2. $\frac{5}{12}\cdot\frac{3}{3}=\frac{15}{36}$

$\frac{2}{9}\cdot\frac{4}{4}=\frac{8}{36}$ Rewrite fractions with common denominator

3. $\frac{15}{36}+\frac{8}{36}=\frac{23}{36}$ Add numerators

To add (or subtract) mixed numbers

1. Write fractions with common denominator.

2. Add the whole numbers.

3. Add the fractions.

4. If the sum of the fractions is more than 1, change the fraction to a mixed number and add again.

EXAMPLE: $11\frac{7}{10}+23\frac{8}{15}$

$11\frac{7}{10}=11\frac{21}{30}$ Write fractions with common denominator

$23\frac{8}{15}=23\frac{16}{30}$

$34\frac{37}{30}$ Add

$34+1\frac{7}{30}$ Change fraction to mixed number

$35\frac{7}{30}$ Add again

FRACTION WORKSHEET: ADDITION AND SUBTRACTION

		Answers
1.	$\frac{3}{5}+\frac{8}{15}$	$\frac{17}{15}$
2.	$\frac{5}{18}+\frac{4}{27}$	$\frac{23}{54}$
3.	$\frac{9}{28}-\frac{2}{21}$	$\frac{19}{84}$
4.	$\frac{8}{10}-\frac{3}{15}$	$\frac{3}{5}$
5.	$\frac{20}{35}-\frac{24}{42}$	0
6.	$\frac{7}{72}-\frac{1}{45}$	$\frac{3}{40}$
7.	$\frac{2}{39}+\frac{1}{3}+\frac{4}{13}$	$\frac{9}{13}$
8.	$\frac{2}{3}-\frac{5}{9}+\frac{1}{12}$	$\frac{7}{36}$
9.	$\frac{9}{16}+\frac{5}{48}+\frac{3}{32}$	$\frac{73}{96}$
10.	$5\frac{3}{4}+7\frac{5}{12}+2\frac{1}{8}$	$15\frac{7}{24}$
11.	$5\frac{2}{7}-3\frac{4}{7}$	$1\frac{5}{7}$
12.	$\frac{3}{10}-\frac{298}{1000}$	$\frac{1}{500}$
13.	$\frac{7}{12}-\frac{3}{20}$	$\frac{13}{30}$

ARITHMETIC REVIEW: WORD PROBLEMS

Basic Strategy for Solving Word Problems:

1. Read the problem carefully until you understand the problem and know what is being asked.
2. If possible, draw a picture or diagram that will help you understand and visualize the problem. Label the parts.
3. Decide what operations (addition, subtraction, multiplication, division, etc.) are necessary to solve the problem.
4. State the problem in mathematical terms.
5. Perform operations.
6. Check your answer and make sure your answer is reasonable.
7. Make sure you have answered the question being asked.

EXAMPLE: In July, Steve opened a checking account and deposited $5850. During the month, he made another deposit of $2500 and wrote checks for $655, $96, $174, and $37. What was his balance at the end of the month?

(1) Add deposits $5850
2500
$8350

(2) Add withdrawals $655
96
174
37
$962

(3) Find the difference:

Deposits - withdrawals = $8350 - $962 = The balance is $7388.00

WORD PROBLEM WORKSHEET

		Answers
1.	At one kennel, 11/18 of the dogs are small, 1/9 are medium size, and the rest are large. What fraction of the dogs are large?	5/18 are large
2.	A rectangle is 3/8 inch by 4/5 inch. Find its area.	3/10 sq. in.
3.	A woman has an estate of $10,000. She leaves 2/5 to a charity. Of the remainder, 2/3 goes to her son. How much money does her son get?	$4000.00
4.	How many 16-ounce cans of beverage can be filled from a vat holding 9280 ounces of the beverage?	580 cans
5.	Mike Johnson worked 16 1/4 hours at $6 per hour. How much money did he earn?	$97.50
6.	Sara worked 15 1/8 hours over the weekend. She worked 6 1/2 hours on Saturday. How many hours did she work on Sunday?	8 5/8 hours
7.	How many horse blankets can be made from 78 3/4 yards of material if each blanket requires 4 3/8 yards?	18 blankets

ARITHMETIC REVIEW: ORDER OF OPERATIONS

Order of Operations:

1. First simplify within parentheses ( ), brackets [ ], and braces { }. Start with the innermost group.

2. Simplify any expressions with exponents and square roots.

3. Multiply or divide from left to right.

4. Add or subtract from left to right.

EXAMPLE: $80 - 24 \div (\underline{3 + 1}) + (\underline{3} \cdot \underline{10}) \div 6 + 4^2$ Remove ( )'s

1. $80 - 24 \div 4 + 30 \div 6 + \underline{4}^2$ Exponents

2. $80 - \underline{24 \div 4} + \underline{30 \div 6} + 16$ Multiplication and Division

3. $80 - 6 + 5 + 16$ Addition and subtraction

4. 95

ORDER OF OPERATIONS WORKSHEET

		Answers
1.	$8 \div 2 + 6 - 5 \cdot 2$	0
2.	$8 \div (2 + 6) + 14 \cdot 2$	29
3.	$2 \cdot 5^2 + 3(16 - 2 \cdot 8) + 5 \cdot 2^2$	70
4.	$\left[\frac{3}{4} - \frac{5}{8}\right] \div \left[\frac{15}{16} - \frac{1}{2}\right]$	$\frac{2}{7}$
5.	$\frac{1}{2} \div \frac{3}{4} + \frac{5}{6} \cdot \frac{1}{5}$	$\frac{5}{6}$
6.	$\frac{9}{10} - \left[\frac{1}{4}\right]^2 + \frac{1}{2}$	$\frac{107}{80}$
7.	$\frac{1}{2} \cdot \frac{4}{5} + \frac{2}{3} \cdot \frac{9}{5}$	$\frac{8}{5}$
8.	$6 + 10 \div 2 + 3 \cdot 3$	20
9.	$6 \cdot 3 - 8 \cdot 2 + 4$	6
10.	$5 \cdot \sqrt{36} - 7(7 - 4)$	9
11.	$6 \cdot \sqrt{25} \cdot \sqrt{100} \div 3 \cdot \sqrt{4 \cdot 9}$	600
12.	$1 + 3 - 2 \cdot \sqrt{1} + 3 \cdot \sqrt{121} - 5 \cdot 3$	20
13.	$3 \cdot 5^2 - 2(14 \div 2 - 7) + 10^3$	1075
14.	$(15 - 10)[(9 + 3^2) \div 2 + 6]$	75
15.	$100 + 2[(7^2 - 9)(5 + 1)^2]$	2980

ARITHMETIC REVIEW: OPERATIONS WITH DECIMALS

Addition and Subtraction of Decimals:

1. Write decimals in columns making sure the decimals are aligned vertically. (Insert extra zeros to align digits)

2. Add the numbers.

3. Place the decimal point in the sum aligned with the other decimal points.

EXAMPLE: 5.8 + 11.03 + 4.1

5.80	Write decimals and add zeros to align digits.
11.03	
4.10	
20.93	Place decimal point in sum.

Multiplication of Decimals:

1. Multiply the two numbers as if they were whole numbers.

2. Count the total number of places to the right of the decimal points in both numbers. The total is the number of decimal places the product must have.

3. Place the decimal point in the product so the number of places to the right of the decimal point is the same as the total.

EXAMPLE: 11.2 x 5.03

112 x 503 = 56336	Multiply
3 places	Count places
56.336	Place decimal 3 places to the left.

Division of Decimals:

1. If the divisor is not a whole number, move the decimal point to the right so the divisor is a whole number.

2. Move the decimal point in the dividend the same number of places (add zeros if necessary).

3. Place a decimal point in the quotient above the decimal point in the dividend.

4. Divide the two numbers as if they were whole numbers.

EXAMPLE: 237.86 ÷ 1.4

$1.4\overline{)237.86}$ Move decimal point.

$$\begin{array}{r} . \\ 14\,\overline{)2378.6} \end{array}$$

Place decimal in quotient

$$\begin{array}{r} 169.9 \\ 14\,\overline{)2378.6} \\ \underline{14} \\ 97 \\ \underline{84} \\ 138 \\ \underline{126} \\ 126 \\ \underline{126} \\ \text{-0-} \end{array}$$

Divide

Answer: 169.9

DECIMAL WORKSHEET

		Answers
1.	0.56 + 3.01 + 1.73	5.3
2.	5.475 + 13.436 + 51.001	69.912
3.	4 + 1.48 + 111 + .0374	116.5174
4.	5.2 - 2.81	2.39
5.	59.7 - 46.39	13.31
6.	52.115 - 19.98	32.135
7.	.4 x 5.2	2.08
8.	0.189 x 3.89	.73521
9.	3.32 x 6.114	20.29848
10.	813.5 ÷ 3.5	232.4285714
11.	10.9 ÷ 1.21	9.008264463
12.	20.912 ÷ .47	44.49361702

ARITHMETIC REVIEW: DECIMALS

Changing a decimal to a fraction:

1. Write the digits after the decimal point as the numerator of the fraction.
2. The denominator is 1 followed by as many zeros as there are digits to the right of the decimal point.
3. Reduce the fraction.

EXAMPLE: .25

$$\frac{25}{100}$$

25 as numerator
1 + 00 (2 decimal points)

$$\frac{\overset{1}{\cancel{25}}\cdot 1}{\underset{1}{\cancel{25}}\cdot 4} = \frac{1}{4}$$

Reduce fraction

Changing a fraction to a decimal:

Divide the numerator of the fraction by the denominator.

EXAMPLE: $\frac{3}{5}$

$$\begin{array}{r} .6 \\ 5\overline{)3.0} \end{array} = .6$$

Terminating

EXAMPLE: $\frac{2}{3}$

$$\begin{array}{r} .666\ldots \\ 3\overline{)2.000} \end{array} = .667$$

Non-terminating
Rounded to 3 decimal places

ARITHMETIC REVIEW: PERCENTS

Changing Fractions to Percents:

1. Change the fraction to a decimal.

2. Multiply by 100 (This results in moving the decimal point two places to the right.)

3. Attach a percent sign - %.

EXAMPLE: $\frac{1}{4}$

$\begin{array}{r} .25 \\ 4\overline{)1.00} \end{array}$	Change to a decimal
.25 x 100 = 25	Multiply by 100
25%	Attach percent sign

Changing Percents to Decimals to Fractions:

1. Drop the percent sign - %.

2. Divide by 100 (This results in moving the decimal point two places to the left.)

3. Change to a fraction and reduce.

EXAMPLE: 15%

15	Drop % sign
15 ÷ 100 = .15	Divide by 100
$\frac{15}{100}$	Change to a fraction
$\frac{15}{100} = \frac{5.3}{5.20} = \frac{3}{20}$	Reduce

ARITHMETIC REVIEW: ORDERING TERMS

Determining Value and Order of Terms:

1. When comparing terms, put all terms in decimal form.

2. Rewrite all terms so they have the same number of decimal places to the right of the decimal point. (Add zeros when necessary.)

3. Write the terms from smallest to largest ignoring the decimal point.

4. Remove extra zeros.

EXAMPLE: .3 , .268 , $\frac{1}{4}$, .681 , $\frac{3}{5}$

.3 , .268 , .25 , .681 , .6	Change to decimals
.300 , .268 , .250 , .681 , .600	Rewrite terms
.250 , .268 , .300 , .600 , .681	Order numbers
.25 , .268 , .3 , .6 , .681	Remove zeros

ARITHMETIC REVIEW: PROPORTIONS

Solving Proportions:

1. In the proportion, the unknown term is the variable. (The value you must find.)
2. Cross multiply and write an equation.
3. Divide to find the missing number.

EXAMPLE: $\frac{5}{9} = \frac{10}{x}$ x is the variable

$5x = 90$ Cross product

$x = \frac{90}{5} = 18$ Divide by 5

Using Proportions to solve Percent problems.

$$\frac{a}{b} = \frac{\%}{100}$$

Where a = part % = part
b = whole 100 = whole

EXAMPLE: What is 15% of 300?

$\frac{a}{300} = \frac{15}{100}$ 300 = whole 15 = %
Find a

$a = 45$

EXAMPLE: 32 is what percent of 80?

$\frac{32}{80} = \frac{\%}{100}$ 32 = part Find %
80 = whole

$\% = 40$

EXAMPLE: 20 is 5% of what number?

$\frac{20}{b} = \frac{5}{100}$ 20 = part
5 = %

$b = 400$ Find b

DECIMAL - FRACTION - PERCENT WORKSHEET

1. Complete this chart. Round decimals to the nearest thousandth and percents to the nearest tenth of a percent.

FRACTION	DECIMAL	PERCENT	ANSWERS
__________	.1	__________	1/10, 10%
__________	__________	12.5%	1/8, .125
3/10	__________	__________	.3, 30%
__________	.4	__________	2/5, 40%
2/3	__________	__________	.667, 66.7%
__________	__________	90%	9/10, .9
__________	2.5	__________	2 1/2, 250%
5/6	__________	__________	.833, 83.3%

2.	What is 12% of 48?	5.76
3.	18 is 72% of what number?	25
4.	52 is what percent of 104?	50%
5.	What percent of 50 is 30?	60%
6.	What is .68% of 487?	3.312
7.	What percent of 190 is 85?	44.7%
8.	Write < or > to make a true statement:	
	3/8 __________ .38	<
	1/4 __________ .28	<
	5/8 __________ .6	>
	7/8 __________ .9	<

MAT 095

Elementary Algebra

BEGINNING ALGEBRA PRE-TEST

1. Add: $\frac{3}{16} + \frac{1}{2}$ 1. ____________

2. Subtract: $2\frac{1}{3} - \frac{5}{6}$ 2. ____________

3. Multiply: $\frac{10}{13} \times \frac{26}{15}$ 3. ____________

4. Divide: $2\frac{4}{5} \div \frac{7}{11}$ 4. ____________

5. Stephen's take home pay is $1200.00 a month. If his rent is $\frac{1}{4}$ of his pay, how much is his rent? 5. ____________

6. Find the least common denominator for the given pair of fractions.
$\frac{7}{10}, \frac{11}{45}$ 6. ____________

7. Write the number 136 as a product of prime factors. 7. ____________

8. A tip of 15% is left after a dinner that costs $54.00. How much is the tip? 8. ____________

9. Convert the decimal 0.002 to a percent. 9. ____________

10. Write the fraction $\frac{23}{50}$ as a percent. 10. ____________

11. Convert 52% to a fraction in lowest term. 11. ____________

12. Write $<$ or $>$ to make a true statement.
$\frac{7}{8}$ ____ 0.8 12. ____________

13. Solve: $\frac{x}{12} = \frac{5}{8}$ 13. ____________

14. Evaluate: $3 \bullet 4^2 - [5 \bullet (9-2)] - 6^2$ 14. ____________

BEGINNING ALGEBRA PRE-TEST - ANSWERS

1. $\frac{11}{16}$

2. $\frac{3}{2}$ *or* $1\frac{1}{2}$

3. $\frac{4}{3}$ *or* $1\frac{1}{3}$

4. $\frac{22}{5}$ *or* $4\frac{2}{5}$

5. $300.00

6. 90

7. $2 \, x \, 2 \, x \, 2 \, x \, 17$

8. $8.10

9. 0.2%

10. 46%

11. $\frac{13}{25}$

12. >

13. $\frac{15}{2}$ *or* $7\frac{1}{2}$

14. - 23

BEGINNING ALGEBRA REVIEW - INTEGERS

Definition: The absolute value of a number can never be negative. The absolute value of a number is the distance between 0 and the number on the number line. The symbol for "The absolute value of a" is $|a|$.

EXAMPLES:
1) $|-4| = 4$
2) $|3| = 3$

To add two signed numbers:

- Like signs: Add their absolute values and use the common sign.

EXAMPLES: 1) $-3 + (-2) = -(3+2)$ $= -5$ 2) $4 + 8 = (4+8)$ $= 12$

- Unlike signs: Subtract their absolute values and use the sign of the number with the larger absolute value.

EXAMPLES: 1) $-3 + 4 = +(4-3)$ $= 1$ 2) $2 + (-6) = -(6-2)$ $= -4$

To subtract two signed numbers:

1) Change the subtraction symbol to addition.
2) Change the sign of the number being subtracted.
3) Use rules for addition

EXAMPLES: 1) $-3 - 4 = -3 + (-4)$ $= -7$ 2) $6 - (-2) = 6 + (2)$ $= 8$

3) $-2 - (-4) = -2 + 4$ $= 2$

Multiplication and division of two signed numbers:

- Like signs: The product or quotient of two numbers with like signs is positive.

EXAMPLES: 1) $(-3)(-4) = 12$ 2) $\frac{8}{4} = 2$ 3) $\frac{-16}{-4} = 4$

- Unlike signs: The product or quotient of two numbers with unlike signs is negative.

EXAMPLES: 1) $(-3)(4) = -12$ 2) $\frac{-15}{3} = -5$

- Division by 0 is undefined.

EXAMPLE: 1) $-\frac{3}{0}$ is undefined.

BEGINNING ALGEBRA REVIEW: ADDITION AND SUBTRACTION OF SIGNED NUMBERS WORKSHEET

		Answers
1.	5 + (-3)	2
2.	- 6 + (-2)	- 8
3.	7 - 12	- 5
4.	- 11 - 4	- 15
5.	6 + [2 + (- 13)]	-5
6.	8 - (- 5)	13
7.	- 3 - (4 - 11)	4
8.	- 5/6 - 1/2	- 4/3
9.	[(- 9) + (- 14)] + 12	- 11
10.	- 4.4 - 8.6	- 13
11.	2 + (- 4 - 8)	- 10
12.	9/10 + (- 3/5)	3/10
13.	- 8 - [(- 4 - 1) - (9 - 2)]	4
14.	[- 8 + (- 3)] + [- 7 + (- 6)]	- 24
15.	- 4 + [(- 12 + 1) - (- 1 - 9)]	-5

BEGINNING ALGEBRA REVIEW: MULTIPLICATION AND DIVISION OF SIGNED NUMBERS WORKSHEET

		Answers
1.	(3) (-4)	- 12
2.	$\frac{24}{-6}$	- 4
3.	(-10) (-12)	120
4.	$\frac{0}{-2}$	0
5.	$\left(-\frac{3}{8}\right)\left(-\frac{10}{9}\right)$	$\frac{5}{12}$
6.	(- 9.8) ÷ (-7)	1.4
7.	$\frac{-30}{2-8}$	5
8.	(-5.1) (.02)	- .102
9.	$\frac{-40}{8-(-2)}$	-4
10.	(- 9 - 1) (-2) - (- 6)	26

BEGINNING ALGEBRA REVIEW: LINEAR EQUATIONS AND INEQUALITIES

Solving Linear Equations:

1. Remove grouping symbols by using the distributive property.
2. Combine like terms to simplify each side.
3. Clear fractions by multiplying through by the Lowest Common Denominator.
4. Move the variable to one side and the constants to the other side. Do this by adding or subtracting terms.
5. Solve for the variable by multiplying by the inverse or dividing by the coefficient of x.
6. Check by substituting the result into the original equation.

EXAMPLES:

1) $3(x - 4) - (2x + 8) = 4x + 4$

$3x - 12 - 2x - 8 = 4x + 4$ Remove grouping symbols

$x - 20 = 4x + 4$ Combine terms

$$\begin{array}{r} x - 20 = 4x + 4 \\ -4x + 20 \quad -4x + 20 \\ \hline -3x = 24 \end{array}$$

Move variable to one side, numbers to the other

$$\frac{-3x}{-3} = \frac{24}{-3}$$

Divide to solve for the variable

$$x = -8$$

$3(-8 -4) - (2(-8) + 8) = 4(-8) + 4$ Check

$3(-12) - (-16 + 8) = -32 + 4$

$-36 - (-8) = -28$

$-28 = -28$

2) $\frac{2}{3}x - \frac{1}{2} = \frac{3}{4}$

$12\left(\frac{2}{3}x\right) - 12\left(\frac{1}{2}\right) = 12\left(\frac{3}{4}\right)$ Clear fractions by multiplying by the LCD

$$\begin{array}{r} 8x - 6 = 9 \\ +6 \quad +6 \\ \hline 8x = 15 \end{array}$$

$$\frac{8x}{8} = \frac{15}{8}$$

$$x = \frac{15}{8}$$

Solving Linear Inequalities:

Linear inequalities are solved almost exactly like linear equations. There is only one exception: if it is necessary to divide or multiply by a <u>negative</u> number, the inequality sign must be reversed.

EXAMPLE:

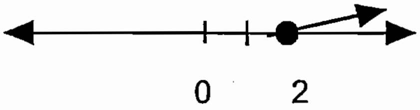
$$-3(x - 2) < x - 5$$
$$-3x + 6 < x - 5$$
$$\underline{-x \quad -6 \quad - \quad x - 6}$$
$$\frac{-4x}{-4} < \frac{-11}{-4}$$

$$x > \frac{11}{4}$$ Reverse inequality because of division by -4

Graphing solutions to inequalities:

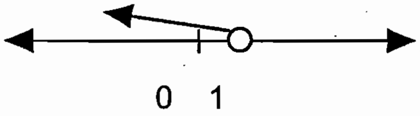
* * Use a solid dot if the endpoint **<u>is</u>** included.

EXAMPLE: $x \geq 2$



* * Use an open circle if the endpoint **<u>is not</u>** included.

EXAMPLE: $x < 1$



BEGINNING ALGEBRA REVIEW: LINEAR EQUATION AND INEQUALITY WORKSHEET

		Answers
1.	$4a - 7 = 3(2a + 5) - 2$	$a = -10$
2.	$2(5x + 3) - 3 = 6(2x - 3) + 15$	$x = 3$
3.	$(\frac{1}{2})(x - 5) = (\frac{1}{3})(x + 2)$	$x = 19$
4.	$10 - 4x + 8 \geq 6 - 2x + 10$	$x \leq 1$
5.	$5(x + 3) - 6x \leq 3(2x + 1) - 4x$	$x \geq 4$
6.	$2(x - 5) + 3x < 4(x - 6) + 3$	$x < -11$

BEGINNING ALGEBRA REVIEW: POLYNOMIALS

Definition: A sum of a finite number of terms of the form: $a_n x^n$ where a_n is a real number and n is a non-negative integer. (No negative exponents, no fractional exponents.)

EXAMPLES: 1) $3x^4 + 2x^3 - 8x + 1$ is a polynomial

2) $2x^{-5} + 3x^2$ is not a polynomial

Types of Polynomials:

- Monomial: a polynomial with 1 term
  EXAMPLE: $3x^4$
- Binomials: A polynomial with 2 terms
  EXAMPLE: $2x^3 + 4$
- Trinomial: A polynomial with 3 terms
  EXAMPLE: $3x^2 + 4x - 2$

Degree:

- Degree of a term - the sum of the exponents on the variables.
  EXAMPLE: $3x^4y^3$ has degree $4 + 3 = 7$
- Degree of a polynomial - the highest term degree. In a polynomial with one variable it is the highest exponent.
  EXAMPLE:
  1) $2x^2y + 3xy - 4$ has degree 3 since $2x^2y$ has degree $2 + 1 = 3$
  2) $5x^4 - 3x^3 + 2x^2 - 8$ has degree 4

Operations with Polynomials:

1. <u>Adding</u> Polynomials: Combine like terms.
   EXAMPLE:
   $(3x^2 + 4x - 5) + (2x^2 - 8x + 2) = 3x^2 + 4x - 5 + 2x^2 - 8x + 2 = 5x^2 - 4x - 3$
2. <u>Subtracting</u> Polynomials: Distribute the negative sign and combine like terms.
   EXAMPLE:
   $(3x^2 + 4x - 5) - (2x^2 - 8x + 2) = 3x^2 + 4x - 5 - 2x^2 + 8x - 2 = x^2 + 12x - 7$
3. <u>Multiplying</u> Polynomials: Use the distributive property.
   EXAMPLE:
   $$\begin{aligned}(3x + 2)(x^2 - 3x + 4) &= 3x(x^2 - 3x + 4) + 2(x^2 - 3x + 4)\\ &= 3x^3 - 9x^2 + 12x + 2x^2 - 6x + 8\\ &= 3x^3 - 7x^2 + 6x + 8\end{aligned}$$
4. <u>Multiplying</u> Binomials: F O I L (<u>f</u>irst, <u>o</u>uter, <u>i</u>nner, <u>l</u>ast)
   EXAMPLE:
   $$\begin{aligned}(2x + 3)(3x - 4) &= (2x)(3x) + (2x)(-4) + 3(3x) + 3(-4)\\ &= 6x^2 - 8x + 9x - 12\\ &= 6x^2 + x - 12\end{aligned}$$

BEGINNING ALGEBRA REVIEW: POLYNOMIAL WORKSHEET

Find the degree of the following and determine what type of polynomial is given.

		Answers
1.	$3x^3y^2 + 2xy - 8$	Trinomial, degree 5
2.	$5 - 2x^3$	Binomial, degree 3
3.	x^5	Monomial, degree 5
4.	8	Monomial, degree 0
Add:		
5.	$(3x^3 + 2x^2 - 5x + 4) + (6x^3 - 2x - 8)$	$9x^3 + 2x^2 - 7x - 4$
Subtract:		
6.	$(3x^3 + 2x^2 - 5x + 4) - (6x^3 - 2x - 8)$	$-3x^3 + 2x^2 - 3x + 12$
Multiply:		
7.	$(x + 2)(2x^2 - 5x + 3)$	$2x^3 - x^2 - 7x + 6$
8.	$(3x + 2)(2x - 3)$	$6x^2 - 5x - 6$
9.	$(3x - 4)^3$	$27x^3 - 108x^2 + 144x - 64$

BEGINNING ALGEBRA REVIEW: EXPONENTS

Definition: $a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ times}}$

a = the base

n = the exponent

Properties:

1. <u>Product Rule</u>: $a^n \cdot a^m = a^{n+m}$

   EXAMPLE: $a^2a^3 = a^{2+3} = a^5$

2. <u>Power Rules</u>: $(ab)^n = a^nb^n$

   EXAMPLE: $(ab)^2 = a^2b^2$

   $$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

   EXAMPLE: $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$

   $$(a^n)^m = a^{nm}$$

   EXAMPLE: $(a^2)^3 = a^{2x3} = a^6$

3. <u>Zero Power Rule</u>: $a^\circ = 1$

4. <u>Negative Exponent Rule</u>: $a^{-n} = \frac{1}{a^n}$

   EXAMPLE: $a^{-4} = \frac{1}{a^4}$

   $$\frac{1}{a^{-n}} = a^n$$

   EXAMPLE: $\frac{1}{a^{-3}} = a^3$

5. <u>Quotient Rule</u>: $\frac{a^n}{a^m} = a^{n-m}$

   EXAMPLE: $\frac{a^5}{a^2} = a^{5-2} = a^3$

Simplifying Exponential Expressions: Possible Steps.

1. Distribute exponents (Power Rules)

2. Clear negative exponents (Negative Exponent Rule)

3. Use the product rule to combine exponents in the numerator and to combine exponents in the denominator.

4. Use the quotient rule to put the exponents in the numerator and the denominator together.

* * Use the zero power rule whenever it is appropriate.

EXAMPLE: $\frac{(xy^2)\,(x^{-2}y)^3}{(xy)^{-1}(xy)^2} = \frac{(xy^2)\,(x^{-6}y^3)}{x^{-1}\,y^{-1}x^2y^2} = \frac{xy^2y^3xy}{x^6x^2y^2}$

$= \frac{x^2y^6}{x^8y^2} = \frac{y^4}{x^6}$

NOTES:

1. The exponent only controls the base.

EXAMPLE: $(-2)^2 = (-2)\,(-2)$ $-2^2 = -(2 \cdot 2)$

$= 4$ $= -4$

Base is -2 Base is 2

2. Negative exponents and negative numbers are not the same.

EXAMPLE: $3^{-1} = \frac{1}{3}$ but $-3 \neq \frac{1}{3}$

BEGINNING ALGEBRA REVIEW: EXPONENT WORKSHEET

		Answers
1.	$(-3)^2$	9
2.	$-\ 3^2$	-9
3.	$3^0 \cdot 3^{-12} \cdot 3^8$	$\frac{1}{3^4}$
4.	$\left[\frac{4 \cdot 5^{-1}}{2^{-3}}\right]^{-2}$	$\frac{5^2}{4^2 2^6}$
5.	$(3a^0b^4)(3ab^3c^2)$	$9\ a\ b^7\ c^2$
6.	$(-3\ e\ b\ c^2)^2\ (2\ a\ b^{-13})$	$\frac{18ac^4e^2}{b^{11}}$
7.	$(x^3y^{-2})^{-3}$	$\frac{y^6}{x^9}$
8.	$\frac{6ab^0c^{-2}}{(-3a)^{-1}bc^{-3}}$	$\frac{-18a^2c}{b}$

BEGINNING ALGEBRA REVIEW: GRAPHING LINEAR EQUATIONS

* * The graph of a linear equation is a straight line.

To draw the graph of a linear equation in two variables:

1. Find the x intercept: replace y with 0 in the given equation and solve for x. Write the point as $(x,0)$.
2. Find the y intercept: replace x with 0 in the given equation and solve for y. Write the point as $(0,y)$.
3. Plot the solutions on a rectangular coordinate system.
4. Draw a straight line that passes through both of the plotted points.
5. As a check, find a third point by choosing a value of x or y that has not been used. All three points should be on the same line.

   (Note: If the x and y intercept are the same value $(0,0)$, you must choose an additional point.)

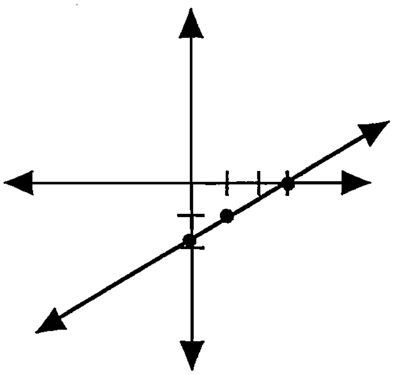
   EXAMPLE: Graph $2x - 3y = 6$

   x intercept: Let $y = 0$
   $2x = 6$
   $x = 3$
   $(3, 0)$

   y intercept: Let $x = 0$
   $-3y = 6$
   $y = -2$
   $(0, -2)$

   Check: Let $x = 1$
   $2(1) - 3y = 6$
   $2 - 3y = 6$
   $-3y = 4$
   $y = -4/3$
   $(1, -4/3)$

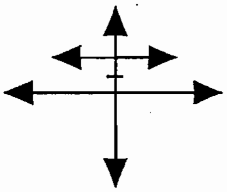
   Graph:



Special Cases:

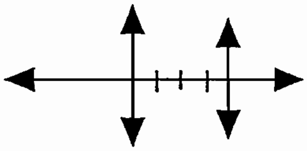
* * The graph of the linear equation $y = k$, where k is a real number, is the <u>horizontal</u> line going through the point (0,k).

EXAMPLE: $y = 2$



* * The graph of the linear equation $x = k$, where k is a real number, is the <u>vertical</u> line going through the point (k,0).

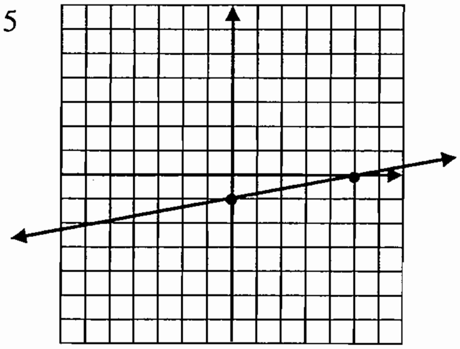
EXAMPLE: $x = 4$



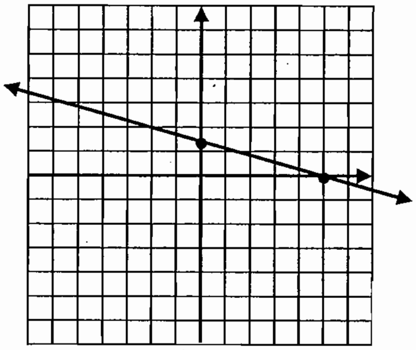
BEGINNING ALGEBRA REVIEW: GRAPHING WORKSHEET

Graph the following equations:

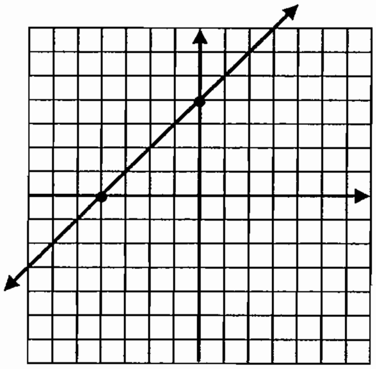
1. $x-5y=5$



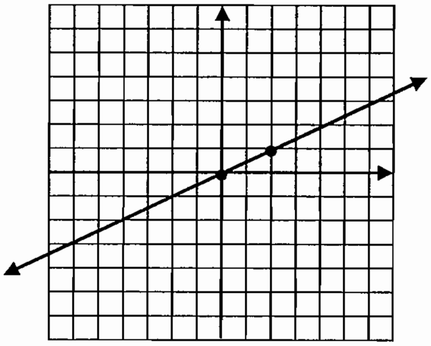
2. $x+3y=5$



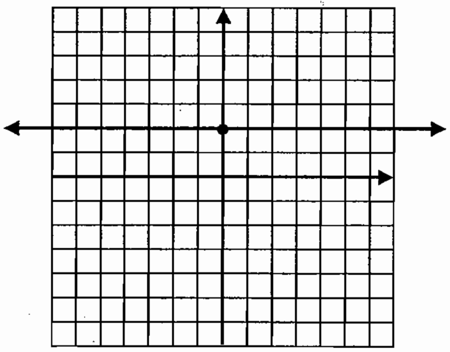
3. $y=x+4$



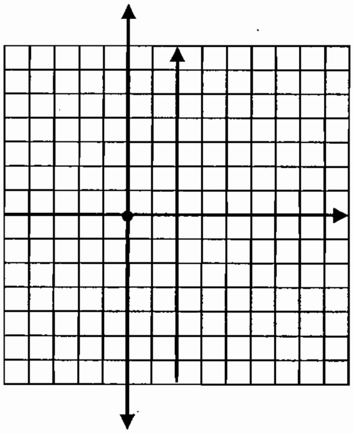
4. $x=2y$



5. $6y=12$



6. $x=-2$



ABSOLUTE VALUE

Definition: $$|X| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

Absolute-Value Principal for Equations:

For $p > 0$, the solutions of $|X| = p$
are those values that satisfy $X = -p$ or $X = p$.
If $|X| = 0$, then $X = 0$.
If $p < 0$, then $|X| = p$ has no solution.

EXAMPLE: $|x - 2| = 3$
$x - 2 = 3$ or $x - 2 = -3$
$x = 5$ $\quad x = -1$

To solve inequalities with absolute-value signs, there are two cases.

a) The solutions of $|X| < p$ are those values that satisfy $-p < x < p$.

EXAMPLE: $|x - 5| < 3$
$-3 < x - 5 < 3$
$2 < x < 8$

b) The solutions of $|X| > p$ are those values that satisfy $X < -p$ or $p < X$.

EXAMPLE: $|x - 7| > 10$
$x - 7 < -10$ or $x - 7 > 10$
$x < -3$ or $x > 17$

Alternate Method

EXAMPLE: 1. $|x - 2| = 3$

$x - 2 = 3 \qquad -(x - 2) = 3$
$x = 5 \qquad -x + 2 = 3$
$-x = 1$
$x = -1$

2. $|x - 5| < 3$

$x - 5 < 3$ and $-(x - 5) < 3$
$x < 8 \qquad -x + 5 < 3$
$-x < -2$
$x > 2$
$2 < x < 8$

3. $|x - 7| > 10$

$x - 7 > 10$ or $-(x - 7) > 10$
$x > 17 \qquad -x + 7 > 10$
$-x > 3$
$x < -3$
$x > 17$ or $x < -3$

ABSOLUTE VALUE WORKSHEET

1. $|x| = 10$
2. $|x| < 13$
3. $|-x| > 2$
4. $|x + 3| = 2$
5. $|9 - 3x| = 0$
6. $|3x - 7| = 5$
7. $|x - 7| < 0$
8. $|2x + 7| < 3$
9. $|4 + 3x| < 10$
10. $|x - 1| > 2$
11. $|7x + 2| > 4$
12. $|x - 2| > -2$
13. $|4x + 2| = 8$
14. $|2x - 3| = 11$
15. $|6x + 1| = 13$
16. $|x| > 12$
17. $|-x| = 15$
18. $|x - 1| = 8$
19. $|3x + 1| = 10$
20. $|x + 3| = -2$
21. $|3x| < 9$
22. $|2x - 5| < 1$

ANSWERS

1. $x = \pm 10$
2. $-13 < x < 13$
3. $x < -2$ or $x > 2$
4. $x = -1$ or -5
5. $x = 3$
6. $x = 4$ or $2/3$
7. No solution
8. $-5 < x < -2$
9. $-14/3 < x < 2$
10. $x > 3$ or $x < -1$
11. $x > 2/7$ or $x < -6/7$
12. All real numbers
13. $x = 3/2$ or $-5/2$
14. $x = 7$ or -4
15. $x = 2, -7/3$
16. $x > 12$ or $x < -12$
17. $x = \pm 15$
18. $x = 9$ or -7
19. $x = 3$ or $-11/3$
20. No solution
21. $-3 < x < 3$
22. $2 < x < 3$

INTERMEDIATE ALGEBRA REVIEW: LINEAR EQUATIONS AND INEQUALITIES

See pages 26 and 27

Special Cases:

1. When your result is a statement without variables (i.e. $2 = 2$ or $0 = 0$), you have <u>infinite solutions</u> and the equation is called an identity

For example:

$$2 + 9x = 3(3x + 1) - 1$$
$$2 + 9x = 9x + 3 - 1$$
$$2 + 9x = 9x + 2$$
$$0 = 0 \qquad \text{Infinite solutions}$$

2. When your result is a statement without variables and is false (i.e. $10 = 2$, or $8 = 0$), you have <u>no solution</u> and the equation is called a contradiction.

For example:

$$3x - 5 = 3(x - 2) + 4$$
$$3x - 5 = 3x - 6 + 4$$
$$3x - 5 = 3x - 2$$
$$-5 = -2 \qquad \text{No solution}$$

LINEAR EQUATIONS AND INEQUALITIES WORKSHEET

		Answers
1.	$3 - (2x + 5) = x - 3(2x - 5)$	$x = 17/3$
2.	$(1/2)(x - 5) = (1/3)(x + 2)$	$x = 19$
3.	$2x + 3 = 3x + 6 - x + 4$	No solution
4.	$2x + 3 = 3x + 6 - x - 3$	Infinite solutions
5.	$3x + 4 - 5x + 2 < 2(3 + x)$	$x > 0$

BEGINNING ALGEBRA REVIEW: COMPOUND LINEAR INEQUALITIES

Compound inequalities are solved the same way as simple inequalities.
Any operation (addition, subtraction, multiplication, division) must be performed on all three pieces of the inequality.

For example:

$$\begin{aligned} -5 &< 3x < 12 \\ (-5/3) &< 3x/3 < 12/3 \\ (-5/3) &< x < 4 \end{aligned}$$

<u>Never</u> remove the variable from the middle piece of the inequality.

COMPOUND INEQUALITIES WORKSHEET

		Answers
1.	$-2 < 3x + 1 < 4$	$-1 < x < 1$
2.	$-5 \leq 4x + 5 \leq 5$	$-5/2 \leq x \leq 0$
3.	$-4 < 4x + 2 \leq 8$	$-3/2 < x \leq 3/2$
4.	$3 < 5 - 2x < 7$	$-1 < x < 1$
5.	$2 \leq 1 - x < 6$	$-5 < x \leq -1$
6.	$-2 < 4 - 3x < 0$	$2 > x > 4/3$
7.	$0 < 1 - 4x < 7$	$-3/2 < x < 1/4$
8.	$-4 \leq 3 - 2x < -1$	$2 \leq x \leq 7/2$
9.	$-7 < 4 - 2x < -4$	$4 < x < 11/2$
10.	$4 < 4 - 3x < 6$	$-2/3 < x < 0$

BEGINNING ALGEBRA REVIEW: LINEAR EQUATIONS

Find the equation of a line.

The standard form of the equation of a line is written as

$$a x + b y = c$$

Where a and b are not both 0

The slope intercept form of the equation of a line is written as

$$y = m x + b$$

Where m is the slope and the point (0, b) is the y-intercept.

The point-slope form of the equation of a line having slope m and passing through the known point (x_1, y_1) is given by

$$y - y_1 = m (x - x_1).$$

Examples:

Find an equation for the following:

1. A line containing (2, -5) and parallel to 3x - 5y = 9

$$3x - 5y = 9$$
$$-5y = -3x + 9$$
$$y = \frac{3}{5}x - \frac{9}{5}$$
$$m = \frac{3}{5}$$
$$y - y_1 = m(x - x_1)$$
$$y + 5 = \frac{3}{5}(x - 2)$$
$$y = \frac{3}{5}x - \frac{31}{5}$$

2. Containing points (4, 5) and (-3, 1)

$$m = \frac{y^1 - y^2}{x_1 - x_2} = \frac{5 - 1}{4 + 3} = \frac{4}{7}$$
$$y - y_1 = m(x - x_1)$$
$$y - 5 = \frac{4}{7}(x - 4)$$
$$y = \frac{4}{7}x + \frac{19}{7}$$

FINDING THE EQUATION OF A LINE: WORKSHEET

Find the equation of the line that satisfies the given conditions.
Write the equation in standard form.

		Answers
1. slope = 1/2	line passes through (3, 4)	x - 2y = -5
2. slope = -5/6	line passes through (0, 0)	5x + 6y = 0
3. slope = 1	y-intercept -3	x - y = 3
4. horizontal line through (1, 4)		y = 4
5. slope is undefined and passing through (-5, 6)		x = -5
6. slope = -3/2	x-intercept -5	3x + 2y = -15
7. horizontal line through (5, -3)		y = -3
8. vertical line passing through (5, -4)		x = 5
9. line passing through (1,2) and (5, 4)		x - 2y = -3
10. line passing through (-3, 4) and (5, -1)		5x + 8y = 17
11. line passing through (4, -3) and (4, -7)		x = 4
12. line parallel to 3 x + y = 6 and passing through (1, 2)		3x + y = 5
13. line passing through (5, -3) and parallel to y - 2 = 0		y = -3
14. line perpendicular to 2x + 5y = 3 and passing through (1, 7)		5x - 2y = -9

BEGINNING ALGEBRA REVIEW: LINEAR SYSTEMS

To solve linear systems of two equations in two variables by elimination:

1. Write the system so that each equation is in standard form.

$$a x + b y = c$$

2. Multiply one equation (or both equations if necessary), by a number to obtain additive inverse coefficients of one of the variables.
3. Add the corresponding members of the resulting equations and solve the new equation in one variable.
4. In step 3, if we get
    a. a false statement; the system is inconsistent; there are no solutions, and the lines are parallel.
    b. the equation $0 = 0$; the system is consistent and dependent; there are infinitely many solutions, and the lines are the same.
5. Substitute the value of the variable into one of the original equations and solve for the other variable.

Examples

1.
$$\begin{array}{r} x + y = 9 \\ \underline{2x - y = -3} \\ \\ 3x = 6 \\ x = 2 \\ 2 + y = 9 \\ y = 7 \\ (2, 7) \end{array}$$

Check: $2 + 7 = 9$
$4 - 7 = -3$

2.
$$\begin{array}{rl} 3x + 2y = 11 & (x\,4) \\ \underline{2x - 8y = -2} & \\ & \\ 12x + 8y = 44 & \\ \underline{2x - 8y = -2} & \\ 14x = 42 & \\ x = 3 & \\ 3(3) + 2y = 11 & \\ 9 + 2y = 11 & \\ 2y = 2 & \\ y = 1 & \\ (3, 1) & \end{array}$$

Check: $3(3) + 2(1) = 11$
$2(3) - 8(1) = -2$

LINEAR SYSTEMS CONTINUED

To solve a linear system of two equations in two variables by substitution:

1. Solve one of the equations for one of the variables (the one with a coefficient of 1 or -1, if this condition exists).
2. Substitute the expression obtained in step 1 for that variable in the other equation, and solve the resulting equation in one variable.
3. In step 1,
    a. if we get 0 = nonzero number a, the solution set is empty, the system is inconsistent, and the lines are parallel.
    b. if we get 0 = 0, the solution set is the solution set of either equation, the system is consistent and dependent, and the lines are the same.
4. Substitute the value for the variable into the equation obtained in step 1 and solve for the other variable.

Examples:

1.

$3x + 5y = 3$
$x = 8 - 4y$

$3 (8 - 4y) + 5y = 3$
$24 - 12y + 5y = 3$
$24 - 7 y = 3$
$-7y = - 21$
$y = 3$
$x = 8 - 4 (3) = -4$
$(-4, 3)$

Check: $3 (-4) +5 (3) = 3$

2.

$4x + 12y = 4$
$-5 x + y = 11$

$4x + 12y = 4$
$y = 5x + 11$
$4x + 12 (5x + 11) = 4$
$4x + 60x + 132 = 4$
$64x = - 128$
$x = -2$
$y = 5 (-2) + 11$
$y = 1$
$(-2, 1)$

Check: $4(-2) + 12 (1) = 4$
$-5(-2) + 1 = 11$

LINEAR SYSTEMS WORKSHEET

		Answers
1.	$x + y = 11$ $x - y = 1$	$x = 6$ $y = 5$
2.	$2x + y = 9$ $3x - y = 11$	$x = 4$ $y = 1$
3.	$2x + y = 5$ $5x - 2y = 8$	$x = 2$ $y = 1$
4.	$3x + 4y = 7$ $8x - y = 7$	$x = 1$ $y = 1$
5.	$2x + 3y = 2$ $10x - 6y = 3$	$x = 1/2$ $y = 1/3$
6.	$2x + 7y = 23$ $x - 4y = -11$	$x = 1$ $y = 3$
7.	$x + y = 3$ $2x + 3y = 8$	$x = 1$ $y = 2$
8.	$5x + y = 5$ $2x + 3y = 2$	$x = 1$ $y = 0$
9.	$4x - 4y = 8$ $x - y = 1$	No solution
10.	$2x - y = 4$ $6x - 3y = 12$	Dependent System

BEGINNING ALGEBRA REVIEW: PROBLEM SOLVING

Strategy for solving word problems:

1. **Read the problem carefully.** It may be necessary to read the problem several times to understand what information has been provided and what question is being asked.
2. **Use diagrams and/or charts** to help organize the given information.
3. **Determine the relationship or formula** that is relevant to the problem. Sometimes the formula is external to the problem, i.e. area, perimeter, interest. If not, then the problem itself provides the required relationship.
4. **Identify the unknown quantity (or quantities)** in terms of one variable and label them, i.e. let x = something.
5. **Write an equation** involving the unknown quantity (quantities).
6. **Solve the equation.**
7. Make sure you have **answered the question** that was asked.
8. **Check your answer(s)** using the original words of the problem.

EXAMPLES:

1. The sum of three times a number and 11 is -13. Find the number.

Let x = the number
$x + 11 = -13$
$x + 11 - 11 = -13 - 11$
$x = -24$
$-24 + 11 = -13$

The number is - 8.

2. Together, a lot and a house cost $40,000. The house costs seven times more than the lot. How much does the lot cost? The house?

Let x = the cost of the lot
7 x = the cost of the house
$x + 7x = 40{,}000$
$8x = 40{,}000$
$x = 5{,}000$
$7x = 35{,}000$
$5{,}000 + 35{,}000 = 40{,}000$

The lot costs $5,000, and the house costs $35,000.

BEGINNING ALGEBRA REVIEW: PROBLEM SOLVING PROBLEMS

1. Five plus three more than a number is nineteen. What is the number?

2. When 18 is subtracted from six times a certain number, the result is 96. What is the number?

3. If you double a number and then add 85, you get three-fourths of the original number. What is the original number?

4. A 180-m rope is cut into three pieces. The second piece is twice as long as the first. The third piece is three times as long as the second. How long is each piece of rope?

5. Lance and Rocky purchased rollerblades for a total of $107. Lance paid $17 more for his rollerblades than Rocky did. What did Rocky pay?

6. A student pays $278 for a calculator and a typewriter. If the calculator costs $64 less than the typewriter, how much did each cost?

BEGINNING ALGEBRA REVIEW: PROBLEM SOLVING SOLUTIONS

1. $5 + (x + 3) = 19$

   $x = 11$ The number is 11.

2. $6x - 18 = 96$

   $x = 19$ The number is 19.

3. $2x + 85 = 3/4x$

   $x = -68$ The number is - 68.

4. $x + 2x + 3(2x) = 180$

   $x = 20$ The lengths are 20 m, 40 m, and 120 m.

5. $x + x + 17 = 107$

   $x = 45$ Lance paid $45.00

6. $x + (x - 64) = 278$

   $x = 171$ The typewriter costs $171.00, and the calculator costs $107.00.

MAT 099

Intermediate Algebra

INTERMEDIATE ALGEBRA
PRE-TEST

Please place correct answer on the right.

Simplify:

1. $\left[\frac{3^2y^{-2}}{4^{-1}y^3}\right]^{-3}$

1. ____________________

Subtract:

2. $6x^3 - 4x^2 + 2$ from $11x^3 + 2x^2 - 8$

2. ____________________

Multiply:

3. $(x+3)\,(x^2 + 5x - 8)$

3. ____________________

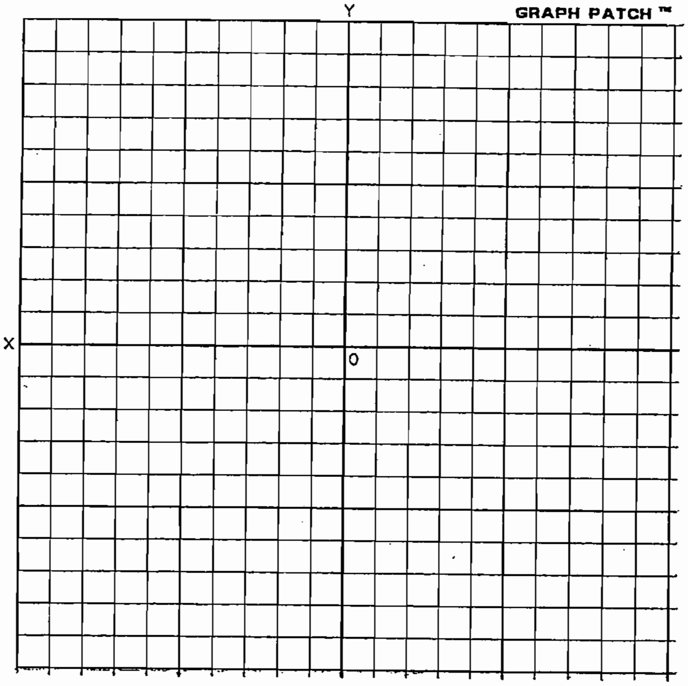
Find the greatest common factor:

4. $9a^4b - 18a^5b + 27a^6b$

4. ____________________

5. Graph:

$4x - 5y = 20$



6. Solve:

$5(x-3) - 7x > 4(x-3) + 9$ and graph the solution set using a number line.

6. ______________________

7. Solve for x $2x - 2(2-3x) = x+3$

7. ______________________

8. Solve for t $\frac{3}{4}t - \frac{7}{12}t = 1$

8. ______________________

9. Solve for x $|2x-5| < 1$

9. ____________________

10. Find the equation of a line passing through $(-3,4)$ *and* $(5,-1)$

10. ____________________

11. Solve the system of equations by method of your choice.

$$\begin{aligned} 2x+3y &= 2 \\ 10x-6y &= 3 \end{aligned}$$

11. ____________________

12. The cost to rent a car, for one day is \$20 plus \$0.25 per mile.

(a) Write a linear equation to compute the cost, y, of driving a car x miles for one day.

12. (a)____________________

(b) Use the equation to compute the cost of driving 258 miles in a rental car.

12 (b)____________________

13. Sketch the region bounded by the following inequalities:

$x \geq 0$
$y \geq, 0$
$x + y \leq 4$

14. Solve for x : $|x - 7| > 10$

14. ____________________

INTERMEDIATE ALGEBRA
PRE-TEST ANSWERS

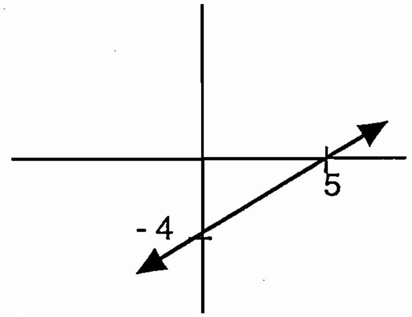
1. $\frac{y^{15}}{4^3 3^6}$

2. $5x^3 + 6x^2 - 10$

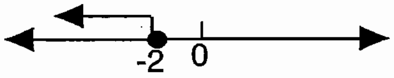
3. $x^3 + 8x^2 + 7x - 24$

4. $9a^4b(1 - 2a + 3a^2)$

5.



6. $x \leq -2$



7. $x = 1$

8. $t = 6$

9. $2 < x < 3$

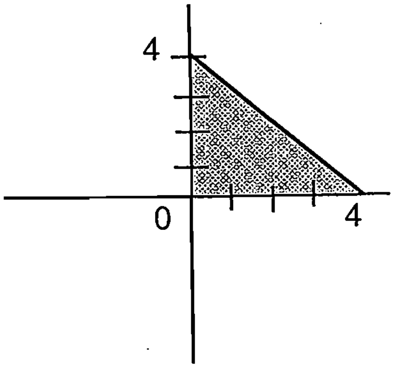
10. $5x + 8y = 17$

11. $x = \frac{1}{2}$; $y = \frac{1}{3}$

12. (a) $y = 0.25x + 20$

(b) $84.50

13.



14. $x > 17$ *or* $x < -3$

INTERMEDIATE ALGEBRA REVIEW: FACTORING

Factoring Steps:

1. If there is a common factor, use the distributive property to factor out the greatest common factor (GCF)

   EXAMPLE: $8x + 32 = 8(x + 4)$

2. Count the terms in the polynomial.

   A. Two terms:

   - If both terms are perfect squares and you have a difference, factor as the difference of two squares.

     $x^2 - y^2 = (x + y)(x - y)$

   EXAMPLE: $x^2 - 9 = (x + 3)(x - 3)$

   - If both terms are perfect cubes, factor as the sum or difference of cubes.

     $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

     $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

   EXAMPLES: 1) $x^3 - 8 = (x - 2)(x^2 + 2x + 4)$

   2) $8x^3 + 1 = (2x + 1)(4x^2 - 2x + 1)$

   (See pages 59 and 60)

   B. Three terms:

   - If the first and last term is a perfect square, try a special product.

     $x^2 \pm 2xy + y^2 = (x \pm y)^2$

   EXAMPLE: $4x^2 + 12x + 9 = (2x + 3)(2x + 3)$

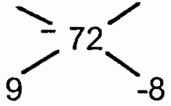
   - If the leading coefficient is 1, factor by trial and error.

   EXAMPLE: $x^2 + 6x + 5 = (x + 5)(x + 1)$

- Split using the method below and factor by grouping:

Method:
1. Multiply first and last coefficients.
2. Find two factors of that product that add up to the middle term.
3. Split the middle term with these two factors.
4. Factor by grouping

EXAMPLE: $6x^2 + x - 12$



$$\begin{aligned} 6x^2 + 9x - 8x - 12 &= (6x^2 + 9x) + (-8x - 12) \\ &= 3x(2x + 3) - 4(2x + 3) \\ &= (3x - 4)(2x + 3) \end{aligned}$$

- Alternate Method: Trial and Error

C. Four or more terms

- Factor by grouping

EXAMPLE:

$$\begin{aligned} 3x^2 + 6x + ax + 2a = (3x^2 + 6x) + (ax + 2a) &= 3x(x + 2) + a(x + 2) \\ &= (3x + a)(x + 2) \end{aligned}$$

3. Check to see if all factors are prime. If not, factor out the GCF.

4. If <u>nothing</u> is done in steps 1 - 3 the polynomial is called <u>prime</u>.

INTERMEDIATE ALGEBRA REVIEW: FACTORING WORKSHEET

		Answers
1.	$a^2 + 17a + 72$	$(a + 8)(a + 9)$
2.	$9m^2 - 64$	$(3m + 8)(3m - 8)$
3.	$2m^2 - 10m - 48$	$2(m + 3)(m - 8)$
4.	$3a^3 + 3ab^2 + 2a^2b + 2b^3$	$(a^2 + b^2)(3a + 2b)$
5.	$8a^2 + 23ab - 3b^2$	$(8a - b)(a + 3b)$
6.	$15x^2 + 11xy - 14y^2$	$(3x - 2y)(5x + 7y)$
7.	$3x^2 + 4x + 2$	Prime
8.	$y^4 - 16$	$(y^2 + 4)(y + 2)(y - 2)$

INTERMEDIATE ALGEBRA REVIEW: FACTORING

See pages 56 and 57

Sums or differences of cubes:

$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$

$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

For Example:

1. $p^3 - 27$, $A = p$, $B = 3$

$(p - 3)(p^2 + 3p + 9)$

2. $x^3 + 125$, $A = x$, $B = 5$

$(x + 5)(x^2 - 5x + 25)$

3. $8y^3 + 27$, $A = 2y$, $B = 3$

$(2y + 3)(4y^2 - 6y + 9)$

FACTORING WORKSHEET

		Answers
1.	$z^3 - 1$	$(z - 1)(z^2 + z + 1)$
2.	$27x^3 + 1$	$(3x + 1)(9x^2 - 3x + 1)$
3.	$64 - 125x^3$	$(4 - 5x)(16 + 20x + 25x^2)$
4.	$x^3 - y^3$	$(x - y)(x^2 + xy + y^2)$
5.	$54x^3 + 2$	$2(27x^3 + 1) = 2(3x + 1)(9x^2 - 3x + 1)$
6.	$ab^3 + 125a$	$a(b^3 + 125) = a(b + 5)(b^2 - 5b + 25)$
7.	$y^3 + 0.125$	$(y + .5)(y^2 - .5y + .25)$
8.	$3z^5 - 3z^2$	$3z^2(z^3 - 1) = 3z^2(z - 1)(z^2 + z + 1)$
9.	$b^3 + \frac{1}{27}$	$(b + \frac{1}{3})(b^2 - \frac{1}{3}b + \frac{1}{9})$
10.	$5x^3 - 40z^3$	$5(x^3 - 8z^3) = 5(x - 2z)(x^2 + 2xz + 4z^2)$

INTERMEDIATE ALGEBRA REVIEW: RATIONAL EXPRESSIONS

To add, subtract, multiply or divide rational expressions, you must factor first and use the following rules for adding, subtracting, multiplying or dividing fractions.

Recall:

1. Multiply two fractions by multiplying the numerators and multiplying the denominators. Use cancellation, if necessary, to write the product in lowest terms.

   EXAMPLE: $\frac{x^2-9}{x+3} \bullet \frac{2x+6}{4} = \frac{(x+3)(x-3)}{\cancel{x+3}} \bullet \frac{\overset{1}{\cancel{2}}(\cancel{x+3})}{\cancel{4}\ 2} = \frac{(x+3)(x-3)}{2}$

2. Divide two fractions by inverting the second fraction (divisor) and multiplying.

   EXAMPLE:

$$\frac{a^2-1}{a-1} - \frac{a^2-2a+1}{a+1} = \frac{a^2-1}{a-1} \bullet \frac{a+1}{a^2-2a+1} = \frac{(a+1)\overset{1}{\cancel{(a-1)}}}{(a-1)} \bullet \frac{a+1}{\underset{1}{(\cancel{a-1})}(a-1)} = \frac{(a+1)^2}{(a-1)^2}$$

3. To add (or subtract) like fractions, add the numerators only and place your result over the common denominator.

   EXAMPLE: $\frac{3}{x+2} + \frac{4x}{x+2} = \frac{3+4x}{x+2}$

4. To add (or subtract) unlike fractions, find the lowest common denominator, rewrite fractions with the common denominator, and add (or subtract) numerators, placing the answer over the common denominator.

   EXAMPLE: $\frac{4}{x^2-4} + \frac{2}{x+2} = \frac{4}{x^2-4} + \frac{2(x-2)}{(x+2)(x-2)} = \frac{4}{x^2-4} + \frac{2x-4}{x^2-4} = \frac{2x}{x^2-4}$

IMPORTANT NOTE: All answers must be simplified, i.e. reduced to lowest terms.

INTERMEDIATE ALGEBRA REVIEW:
RATIONAL EXPRESSIONS WORKSHEET

		Answers
1.	$\frac{x}{x+2}-\frac{4}{x+2}$	$\frac{x-4}{x+2}$
2.	$\frac{3x}{x+3}+\frac{9}{x+3}$	3
3.	$\frac{4}{x+2}+\frac{x}{2x+4}$	$\frac{8+x}{2(x+2)}$
4.	$\frac{3}{x+2}+\frac{4}{x-1}$	$\frac{7x+5}{(x+2)(x-1)}$
5.	$\frac{2}{x-3}-\frac{4}{x-1}$	$\frac{-2x+10}{(x-3)(x-1)}$
6.	$\frac{5}{x^2-4}+\frac{4}{x+2}$	$\frac{4x-3}{(x+2)(x-2)}$
7.	$\frac{3}{x-1}-\frac{8}{1-x}$	$\frac{11}{x-1}$
8.	$\frac{10}{3x+3y}+\frac{13}{2x+2y}$	$\frac{59}{6(x+y)}$
9.	$\frac{x}{x-3}+\frac{4}{x^2-x-6}$	$\frac{x^2+2x+4}{(x-3)(x+2)}$

RATIONAL EXPRESSIONS

See page 61

Examples:

1. $$\frac{x+2}{x-3}\cdot\frac{x^2-4}{x^2+x-2}$$

$$\frac{\cancel{x+2}}{x-3}\cdot\frac{(x+2)(x-2)}{\cancel{(x+2)}(x-1)}$$

$$\frac{(x+2)(x-2)}{(x-3)(x-1)}$$

2. $$\frac{1-a^3}{a^2}\cdot\frac{a^5}{a^2-1}$$

$$\frac{\overset{-1}{\cancel{(1-a)}}(1+a+a^2)}{\cancel{a^2}}\cdot\frac{\overset{a^3}{\cancel{a^5}}}{(a+1)\cancel{(a-1)}}$$

$$\frac{-1\left(1+a+a^2\right)a^3}{a+1}$$

3. $$\frac{a^2-1}{a-1}\div\frac{a^2-2a+1}{a+1}$$

$$\frac{a^2-1}{a-1}\cdot\frac{a+1}{a^2-2a+1}$$

$$\frac{(a+1)\cancel{(a-1)}}{\cancel{a-1}}\cdot\frac{a+1}{(a-1)(a-1)}$$

$$\frac{(a+1)(a+1)}{(a-1)(a-1)}$$

4. $$\frac{2}{21x}+\frac{5}{3x^2}$$

$$\frac{2x}{21x^2}+\frac{35}{21x^2}=\frac{2x+35}{21x^2}$$

5. $$\frac{4x+5}{x+3}-\frac{x-2}{x+3}$$

$$\frac{4x+5-(x-2)}{x+3}$$

$$\frac{4x+5-x+2}{x+3}=\frac{3x+7}{x+3}$$

6. $$\frac{5x}{x-2y}-\frac{3y-7}{2y-x}$$

$$\frac{5x}{x-2y}+\frac{3y-7}{x-2y}=\frac{5x+3y-7}{x-2y}$$

7. $$\frac{2y+1}{y^2-7y+6}-\frac{y+3}{y^2-5y-6}$$

$$\frac{2y+1}{(y-6)(y-1)}-\frac{y+3}{(y-6)(y+1)}$$

$$\frac{(2y+1)(y+1)}{(y-6)(y-1)(y+1)}-\frac{(y+3)(y-1)}{(y-6)(y+1)(y-1)}$$

$$\frac{(2y+1)(y+1)-(y+3)(y-1)}{(y-6)(y-1)(y+1)}$$

$$\frac{2y^2+3y+1-(y^2+2y-3)}{(y-6)(y-1)(y+1)}$$

$$\frac{2y^2+3y+1-y^2-2y+3}{(y-6)(y-1)(y+1)}=\frac{y^2+y+4}{(y-6)(y-1)(y+1)}$$

INTERMEDIATE ALGEBRA REVIEW: RATIONAL EXPONENTS

The rules for exponents apply to <u>all</u> exponents (whole number, positive and negative integers, and rational numbers). (See pages 31 and 32)

In addition to selecting the correct rule for working with exponents, you must also appropriately add, subtract, or multiply fractions.

See the following examples:

1. $5^{1/3} \cdot 5^{1/3} = 5^{1/3 + 1/3} = 5^{2/3}$

2. $3^{1/2} \cdot 3^{1/3} = 3^{1/2 + 1/3} = 3^{3/6 + 2/6} = 3^{5/6}$

3. $\dfrac{6^{1/2}}{6^{1/4}} = 6^{1/2 - 1/4} = 6^{2/4 - 1/4} = 6^{1/4}$

4. $(x^{4/3})^{1/2} = x^{4/3\ 1/2} = x^{2/3}$

5. $(2^3 a^{15} b^{21})^{1/3} = (2^3)^{1/3}\ (a^{15})^{1/3}\ (b^{21})^{1/3}$

   $= 2\ a^5\ b^7$

6. $$\frac{x^{-1/4} y^{2/5}}{x^{3/4} y^{-4/5}} = x^{-1/4-3/4} \bullet y^{2/5-(-4/5)}$$

   $$= x^{-1}\ y^{6/5}$$

   $$= \frac{y^{6/5}}{x}$$

RATIONAL EXPONENTS WORKSHEET

		Answers
1.	$a^{1/3} \cdot a^{2/3}$	a
2.	$x^{1/2} \cdot x^{5/4}$	$x^{7/4}$
3.	$5^{3/2} \cdot 5^{-1/2}$	5
4.	$(a^{2/3})^{4/5}$	$a^{8/15}$
5.	$(x^{3/4})^{4}$	x^{3}
6.	$(a^{-3/4})^{-1/3}$	$a^{1/4}$
7.	$(16y^{4})^{3/4}$	$8y^{3}$
8.	$(a^{3}b)^{2/3}$	$a^{2}b^{2/3}$
9.	$\frac{xy^{3/4}}{x^{1/2}y^{1/4}}$	$x^{1/2}y^{1/2}$

RATIONAL EXPRESSIONS WORKSHEET

		Answers
1.	$\frac{4x^2 - 32x + 60}{2x^2 - 3x - 9}$	$\frac{4(x - 5)}{2x + 3}$
2.	$\frac{x^2}{x + 2} \bullet \frac{x^2 + 4x + 4}{x}$	$x (x + 2)$
3.	$\frac{2x - 12}{20x} \bullet \frac{8x^3}{3x - 18}$	$\frac{4x^2}{15}$
4.	$\frac{x + y}{x - y} \div \frac{3x + 3y}{4x - 4y}$	4/3
5.	$\frac{2x^2 + 3x - 2}{x^2 - x - 20} \bullet \frac{x^2 + x - 12}{x^2 + 3x + 2}$	$\frac{(2x - 1)(x - 3)}{(x - 5)(x + 1)}$
6.	$\frac{x^2 + 7x + 12}{x - 5} \bullet \frac{x^2 - 7x + 10}{x^2 + 6x + 8}$	$\frac{(x + 3)(x - 2)}{x + 2}$
7.	$\frac{x^2 + 4x + 4}{x^2 - 4x + 4} \div \frac{(x + 2)^2}{(x - 2)^2}$	1
8.	$\frac{x - 3}{x^2 + 2x - 3} \bullet \frac{x^2 - 2x + 1}{x^2 - 2x - 3} \div \frac{x^2 - 9}{x^2 - 1}$	$\frac{(x - 1)^2}{(x + 3)^2 (x - 3)}$

NOTE: To simplify rational expressions, factor the numerator and the denominator first and then use cancellation to write the rational expression in lowest terms.

RATIONAL EXPRESSIONS WORKSHEET

		Answers
1.	$\frac{3x + 2}{14} - \frac{5x + 1}{10}$	$\frac{-20x + 3}{70}$
2.	$\frac{10}{3x + 3y} + \frac{13}{2x + 2y}$	$\frac{59}{6(x + y)}$
3.	$\frac{x + 4}{7x - 21} - \frac{x - 4}{7x + 21}$	$\frac{2x}{(x - 3)(x + 3)}$
4.	$\frac{x - 2y}{x^2 - y^2} + \frac{1}{x + y}$	$\frac{2x - 3y}{(x + y)(x - y)}$
5.	$\frac{x^2}{x + 2} - \frac{4}{x + 2}$	$x - 2$
6.	$\frac{5}{2x} + \frac{1}{5x^2} - \frac{3}{10x}$	$\frac{2(11x + 1)}{10x^2}$
7.	$\frac{5}{x^2 - 4} + \frac{4}{x + 2}$	$\frac{4x - 3}{(x + 2)(x - 2)}$
8.	$\frac{6x}{x^2 - x - 6} + \frac{x}{x^2 - 9}$	$\frac{7x^2 + 20x}{(x - 3)(x + 2)(x + 3)}$
9.	$\frac{x^2}{x - 1} + \frac{3x}{4x + 8}$	$\frac{4x^3 + 11x^2 - 3x}{4(x - 1)(x + 2)}$

INTERMEDIATE ALGEBRA REVIEW: RADICALS

Definition: A square root of a number is one of the two equal factors of the number.

EXAMPLE: Square roots of 25 are + 5 and - 5

The symbol $\sqrt{\ }$ for the principal or positive square root, is called a radical sign.

EXAMPLE: $\sqrt{25} = 5$

$\sqrt{n}$ is the positive number whose square is n

Note: If "a" is a negative real number $\sqrt{a}$ is not a real number.

EXAMPLE: $\sqrt{-4}$ is not real

Properties:

1. Product Rule: $\sqrt{a}\sqrt{b} = \sqrt{ab}$ *and* $\sqrt{ab} = \sqrt{a}\sqrt{b}$

   EXAMPLES: 1) $\sqrt{3}\sqrt{2} = \sqrt{6}$

   2) $\sqrt{8} = \sqrt{4}\sqrt{2}$

   $= 2\sqrt{2}$

2. Quotient Rule: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ and $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

   EXAMPLES: 1) $\sqrt{\frac{9}{4}} = \frac{3}{2}$ 2) $\frac{\sqrt{8}}{\sqrt{32}} = \sqrt{\frac{8}{32}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

** For a real number a, $\sqrt{a^2}$ = a only if a is non-negative.

** $\sqrt{a}\sqrt{a} = (\sqrt{a})^2 = a$ *and* $-\sqrt{a}\,-\sqrt{a} = (-\sqrt{a})^2 = a$

ALGEBRA REVIEW: RADICAL WORKSHEET

Use the product rule and the quotient rule to simplify each radical.

		Answers
1.	$\sqrt{45}$	$3\sqrt{5}$
2.	$\sqrt{64}$	8
3.	$\sqrt{75}$	$5\sqrt{3}$
4.	$10\sqrt{27}$	$30\sqrt{3}$
5.	$\sqrt{50}\ \sqrt{20}$	$10\sqrt{10}$
6.	$\sqrt{12}\ \sqrt{48}$	24
7.	$\frac{\sqrt{72}}{\sqrt{8}}$	3
8.	$\frac{15\sqrt{10}}{5\sqrt{2}}$	$3\sqrt{5}$
9.	$\sqrt{x^4}$	x^2
10.	$\sqrt{y^3}$	$y\sqrt{y}$
11.	$\sqrt{x^4y^8}$	x^2y^4
12.	$\sqrt{\frac{16}{x^2}}$	$\frac{4}{x}$
13.	$\sqrt{\frac{100}{m^4}}$	$\frac{10}{m^2}$
14.	$\sqrt{\frac{75}{y^6}}$	$\frac{5\sqrt{3}}{y^3}$

INTERMEDIATE ALGEBRA REVIEW: FUNCTIONS

Definition: A function is a rule that relates x and y so that each appropriate value of x leads to exactly one value of y.

Definition: The collection of all values of x in a function is called the domain. For example, the domain of the function $y = 5x$, where x represents the number of hours you work is $x > 0$. That is, x can't be negative since you cannot work a negative number of hours.

Find the domain: $y = \frac{5}{x + 3}$ $x \neq -3$

$y = x - 2$ x is any real number

Definition: The collection of all possible values for y in a function is called the range. For example, the range of the function $y = 5x$, where y represents your total earnings, is $y > 0$. That is, your total earnings, y, can't be negative.

Find the range: $y = x^2$ $y \geq 0$

$y = x$ y is any real number

Some rules that relate y and x are not functions. For example, $x = y^2$. When $x = 4$, $y = \pm 2$. (i.e. x leads to 2 y-values.)

Vertical Line Test: A graph in the plane represents a function if no vertical line intersects the graph at more than one point.

Notation: To denote that y is a function of x, we write $y = f(x)$. The expression "f(x)" is read "f of x". It does not mean f times x. Since y and f(x) are equal, they can be used interchangeably. This means we can write $y = x^2$, or we can write $f(x) = x^2$.

Evaluation: To evaluate or calculate a function, replace the x in the function rule by the given x value from the domain and then compute according to the rule.

For example: 1. $f(x) = 6x + 5$

$f(2) = 6(2) + 5 = 17$

$f(0) = 6(0) + 5 = 5$

$f(-1) = 6(-1) + 5 = -1$

2. $g(x) = 3x^2 - 5x + 8$

$g(1) = 3(1)^2 - 5(1) + 8 = 6$

$g(0) = 3(0)^2 - 5(0) + 8 = 8$

$g(-1) = 3(-1)^2 - 5(-1) + 8 = 16$

FUNCTIONS WORKSHEET

1. Determine whether or not each relation defines a function.
If not, explain why not.

	Answers
a. $\{ (1, 3), (2, -4), (1, 0) \}$	a. No
b. $\{ (4, 3), (-2, 3), (1,3) \}$	b. Yes
c. $2x + y = 3$	c. Yes
d. $x = -7$	d. No

2. Determine the domain and range of each of the following:

a. $\{ (-3,4), (4,2), (0,0), (-2, 7) \}$	Domain: Range:	$\{ -3, -2, 0, 4 \}$ $\{ 0, 2, 4, 7 \}$
b. $y = 2x + 7$	Domain: Range:	All real numbers All real numbers
c. $y = 3 / (x - 4)$	Domain: Range:	$x \neq 4$ $y \neq 0$
d. $y = \sqrt{x + 2}$	Domain: Range:	$x \geq -2$ $y \geq 0$

3. Given $f(x) = x + 2$ and $g(x) = x - 3$, find the following:

a. $f(-2)$	0
b. $f(-4)$	-2
c. $g(0)$	-3
d. $f(3) - g(2)$	4

INTERMEDIATE ALGEBRA REVIEW: QUADRATIC EQUATIONS

Given the quadratic equation in standard form, $ax^2 + bx + c = 0$,

1. Factor, if possible, the polynomial $ax^2 + bx + c$.
   Set each factor $= 0$, and solve for x.

2. If factoring is difficult or not possible, use the quadratic formula.
   Identify the numerical values of a, b, and c.
   Substitute these values into the formula.
   Simplify the resulting expression.

The Quadratic Formula

If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where a is the coefficient of the second-degree term, b is the coefficient of the first-degree term, and c is the constant.

Examples:

1. $x^2 - 2x - 3 = 0$

$a = 1, \quad b = -2, \quad c = -3$

$$x = \frac{2 \pm \sqrt{4 + 12}}{2}$$

$$= \frac{2 \pm \sqrt{16}}{2}$$

$$= \frac{2 + 4}{2} = 3 \quad \text{or} \quad \frac{2 - 4}{2} = -1$$

2. $5x^2 - 8x = 3$

$5x^2 - 8x - 3 = 0$

$a = 5, \quad b = -8, \quad c = -3$

$$x = \frac{8 \pm \sqrt{64 + 60}}{10}$$

$$= \frac{8 \pm \sqrt{124}}{10}$$

$$= \frac{8 \pm 2\sqrt{31}}{10} = \frac{4 \pm \sqrt{31}}{5}$$

3. $x^2 + x + 1 = 0$

$a = 1, \quad b = 1, \quad c = 1$

$$x = \frac{-1 \pm \sqrt{1 - 4}}{2}$$

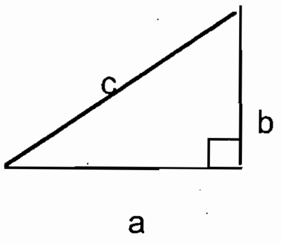
$$= \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}\, i}{2}$$

QUADRATIC EQUATIONS WORKSHEET

		Answers
1.	$x^2 + 3x - 28 = 0$	$x = -7, 4$
2.	$x^2 - x - 12 = 0$	$x = 4, -3$
3.	$3x^2 - 3x - 18 = 0$	$x = 3, -2$
4.	$x^2 - 36 = 0$	$x = \pm 6$
5.	$x^2 = 5x$	$x = 0, 5$
6.	$6x^2 - 5x = 6$	$x = -2/3,\ 3/2$
7.	$x^2 - 30 = x$	$x = 6, -5$
8.	$x^2 + 10x + 4 = 0$	$-5 \pm \sqrt{21}$
9.	$x^2 - 8x - 5 = 0$	$4 \pm \sqrt{21}$
10.	$x^2 + 4x = 1$	$2 \pm \sqrt{5}$
11.	$x^2 + x + 1 = 0$	$\frac{-1 \pm \sqrt{3}\,i}{2}$
12.	$3x^2 - 2x = 4$	$\frac{1 \pm \sqrt{13}}{3}$
13.	$2x^2 = 5 - x$	$\frac{-1 \pm \sqrt{41}}{4}$
14.	$1 = 5x^2 + 7x$	$\frac{-7 \pm \sqrt{69}}{10}$
15.	$x^2 - 11x - 1 = 0$	$\frac{11 \pm \sqrt{5}}{2}$

INTERMEDIATE ALGEBRA REVIEW: PYTHAGOREAN THEOREM

In a right triangle the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the lengths of the legs (the sides that form the right angle). If c is the hypotenuse and a and b are the lengths of the legs, this property can be stated as:



$c^2 = a^2 + b^2$ or $c = \sqrt{a^2 + b^2}$

Also, $a^2 = c^2 - b^2$ or $a = \sqrt{c^2 - b^2}$

$b^2 = c^2 - a^2$ or $b = \sqrt{c^2 - a^2}$

Examples:

1. $a = 3, \quad b = 5$

$c = \sqrt{9 + 25}$

$= \sqrt{34}$

2. $b = 12, \quad c = 13$

$a = \sqrt{169 - 144}$

$= \sqrt{25} = 5$

3. $c = 6, \quad a = \sqrt{5}$

$b = \sqrt{36 - 5}$

$= \sqrt{31}$

PYTHAGOREAN THEOREM WORKSHEET

Find the length of the unknown side in the following right triangles.

1. a = 3 ft. b = 4 ft.

2. b = 16 m. c = 20 m.

3. a = 6 yd. c = 10 yd.

4. A 17-foot ladder is placed against the wall of a house. If the bottom of the ladder is 8 feet from the house, how far from the ground is the top of the ladder?

5. Find the diagonal of a rectangle whose length is 8 meters and whose width is 6 meters.

6. Find the width of a rectangle whose diagonal is 13 feet and length is 12 feet.

ANSWERS

1. c = 5 ft.
2. a = 12 m.
3. b = 8 yd.
4. 15 ft.
5. 10 m.
6. 5 ft.

INTERMEDIATE ALGEBRA REVIEW: RADICALS

In general, the number c is the nth root of a if $c^n = a$. Symbolically, we write

$$c = \sqrt[n]{a}$$

The product and quotient rules hold for all $n \geq 2$.

i.e. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ and $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad \text{and} \quad \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

Examples:

1. $\sqrt[3]{128} = \sqrt[3]{64} \cdot \sqrt[3]{2} = 4\sqrt[3]{2}$

2. $\sqrt[4]{80} = \sqrt[4]{16} \cdot \sqrt[4]{5} = 2\sqrt[4]{5}$

3. $$\begin{aligned}\sqrt[3]{(x+y)^5} &= \sqrt[3]{(x+y)^3} \cdot \sqrt[3]{(x+y)^2} \\ &= (x+y) \cdot \sqrt[3]{(x+y)^2}\end{aligned}$$

4. $$\begin{aligned}\sqrt[5]{x^{11}y^7z^{20}} &= \sqrt[5]{x^5x^5x^1y^5y^2z^5z^5z^5z^5} \\ &= x \cdot x \cdot y \cdot z \cdot z \cdot z \cdot z \ \sqrt[5]{x^1y^2} \\ &= x^2yz^4 \ \sqrt[5]{xy^2}\end{aligned}$$

INTERMEDIATE ALGEBRA REVIEW:
ADDITION / SUBTRACTION OF RADICALS

To have like radicals, the following must be true:

1. The radicals must have the same index.
2. The radicands must be the same.

To combine like radicals (i.e. add or subtract):

1. Perform any simplification within the terms.
2. Use the distributive property to combine terms that have like radicals.

Examples:

1. $6\sqrt{3} + 2\sqrt{3} = (6+2)\sqrt{3} = 8\sqrt{3}$

2. $14\sqrt[5]{2} - 6\sqrt[5]{2} = (14-6)\ \sqrt[5]{2} = 8\sqrt[5]{2}$

3. $2\sqrt{6} + 8\sqrt{6} - 3\sqrt{6} = (2+8-3)\sqrt{6} = 7\sqrt{6}$

4. $8\sqrt{27} - 3\sqrt{3} = 8\sqrt{9.3} - 3\sqrt{3} = 8 \bullet 3\sqrt{3} - 3\sqrt{3} = (24-3)\ \sqrt{3} = 21\sqrt{3}$

5. $9\sqrt{12} + 16\sqrt{27} = 9\sqrt{4 \bullet 3} + 16\sqrt{9 \bullet 3} = (9 \bullet 2)\sqrt{3} + (16.3)\sqrt{3} = (18+48)\sqrt{3} = 66\sqrt{3}$

6. $4\sqrt{3x^3} - \sqrt{12x} = 4\sqrt{3 \cdot x^2 \cdot x} - \sqrt{4 \cdot 3 \cdot x} = 4x\sqrt{3x} - 2\sqrt{3x} = (4x-2)\sqrt{3x}$

7. $\sqrt[3]{54x} - \sqrt[3]{2x^4} = \sqrt[3]{27 \cdot 2 \cdot x} - \sqrt[3]{2 \cdot x^3 \cdot x} = 3\sqrt[3]{2x} - x\sqrt[3]{2x} = (3-x)\sqrt[3]{2x}$

RADICALS WORKSHEET

		Answers
1.	$\sqrt{5} - 3\sqrt{5} + 6\sqrt{5}$	$4\sqrt{5}$
2.	$5\sqrt{2} - 6\sqrt{2} + 10\sqrt{2}$	$9\sqrt{2}$
3.	$2\sqrt{45} - 5\sqrt{20}$	$-4\sqrt{5}$
4.	$5\sqrt{18} - 3\sqrt{32}$	$3\sqrt{2}$
5.	$8\sqrt{5} - \sqrt{20}$	$6\sqrt{5}$
6.	$\sqrt{8x} + \sqrt{32}$	$6\sqrt{2x}$
7.	$\sqrt{28x^3} + \sqrt{63x^3}$	$5x\sqrt{7x}$
8.	$\sqrt{72xy} - \sqrt{200xy}$	$-4\sqrt{2xy}$
9.	$\sqrt{54x^2y} - \sqrt{24x^2y}$	$x\sqrt{6y}$
10.	$\sqrt{100x^2y^2} - \sqrt{81x^2y^2}$	xy

INTERMEDIATE ALGEBRA REVIEW: PROBLEM SOLVING

1. An experienced carpenter can panel a room 3 times faster than an apprentice can. Working together they can panel the room in 6 hours. How long would it take each person working alone to do the job?

2. A company is planning to manufacture computer disks. The fixed cost will be \$60,000 and it will cost \$200 to produce each disk. Each disk will be sold for \$450.

   a) Write the cost function, C, of producing x disks.

   b) Write the revenue function, R, from the sale of x disks.

   c) Determine the break-even point. Describe what this means.

3. In one year, a student earned \$100 interest on money she deposited at a savings and loan. She later learned that the money would have earned \$120 if she had deposited it at a credit union because the credit union paid 1% more interest at the time. Find the rate she received from the savings and loan.

4. The Leaning Tower of Pisa is 176 feet high. The function:

   $$s(t) = -16t^2 + 96t + 176$$

   models the ball's height above the ground, s(t), in feet, t seconds after it was thrown. During which time period will the ball's height exceed that of the tower?

5. Police use the function, $f(x) = \sqrt{20x}$ to estimate the speed of a car, $f(x)$, in miles per hour, based on the length, x, in feet, of its skid marks upon sudden braking on a dry asphalt road.

   A motorist is involved in an accident. A police officer measures the car's skid marks to be 245 feet long. Estimate the speed at which the motorist was traveling before braking. If the posted speed limit is 50 miles per hour and the motorist tells the officer he was not speeding should the officer believe him?

6. A baseball is thrown straight up from a height of 64 feet. The function

   $$s(t) = -16t^2 + 48t + 64$$

   describes the ball's height above the ground s(t), in feet, t seconds after it was thrown. How long will it take for the ball to hit the ground?

7. Use the compound interest formula

$$A = P(1+ r)^t$$

Find the annual interest rate, r, in 2 years when an investment of $3125 grows to $3360.

8. The function $f(x) = - x^2 + 46x - 360$ models the daily profit, f(x), in hundreds of dollars, for a company that manufactures x computers daily. How many computers should be manufactured each day to maximize profit? What is the maximum profit?

9. A tree is supported by a wire anchored in the ground 15 feet from its base. The wire is 4 feet longer than the height that it reaches on the tree. Find the length of the wire.

10. A painting measuring 10 inches by 16 inches is surrounded by a frame of uniform width. If the combined area of the painting and frame is 280 square inches, determine the width of the frame.

INTERMEDIATE ALGEBRA REVIEW: PROBLEM SOLVING
ANSWERS

1. The experienced carpenter can panel a room in 8 hours and the apprentice in 24 hours, working alone.

2. a) $C(x) = 60{,}000 + 200x$

   b) $R(x) = 450x$

   c) (240, 108,000) Revenue generated = cost of producing 240 disks

3. 5%

4. The ball's height exceeds that of the tower between 0 and 6 seconds.

5. speed = 70 mph No

6. $t = 4$ seconds

7. $r = 3.69\%$

8. 23 computers, $16,900

9. 30.125 feet

10. The width of the frame is 2 inches.