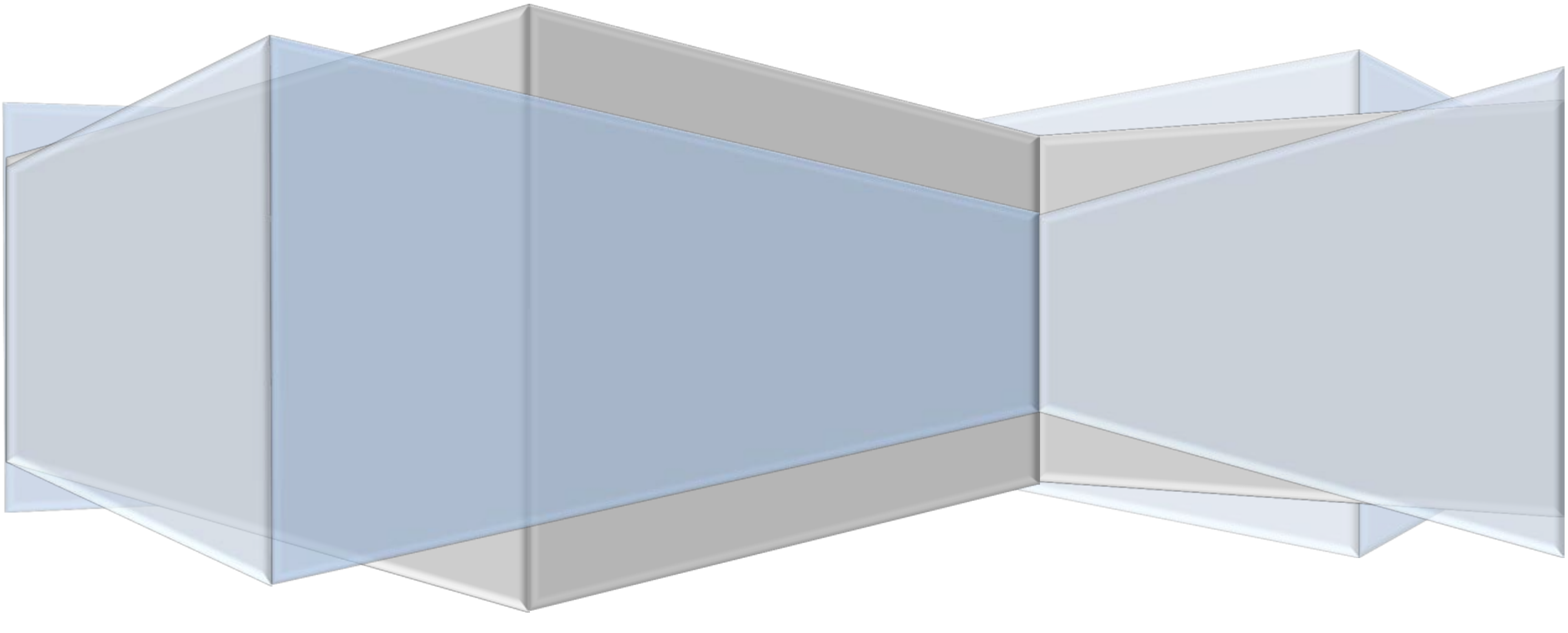
McHenry County College

Review of Intermediate Algebra Content



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FACTORING

FACTORING OUT THE GREATEST COMMON FACTOR

1. Identify the TERMS of the polynomial.
2. Factor each term to its prime factors.
3. Look for common factors in all terms.
4. Factor these factors out.
5. Check by multiplying.

Examples: Factor $6x^2+8xy^2-4x$ — Terms: $6x^2$, $8xy^2$, and $-4x$

$2\cdot 3\cdot x\cdot x+2\cdot 2\cdot 2\cdot x\cdot y\cdot y-2\cdot 2\cdot x$ Factor into primes

$\enclose{circle}{2}\cdot 3\cdot \enclose{circle}{x}\cdot x+\enclose{circle}{2}\cdot 2\cdot 2\cdot \enclose{circle}{x}\cdot y\cdot y-2\cdot \enclose{circle}{2}\cdot \enclose{circle}{x}$ Look for common factors

Answer: $\mathbf{2x\left(3x+4y^2-2\right)}$ Factor out the common factor

Check: $2x(3x)+2x\left(4y^2\right)-2x(2)$ Check by multiplying

$6x^2+8xy^2-4x$

FACTORING TRINOMIALS OF THE FORM: x^2+bx+c

To factor x^2+5x+6, look for two numbers whose product = 6, and whose sum = 5.

List factors of 6:

$1\cdot 6$

$-1\cdot -6$

$\enclose{circle}{2\cdot 3}$

$-2\cdot -3$

Choose the pair which adds to 5

_____ · _____ = 6

_____ + _____ = 5

Since the numbers 2 and 3 work for both, we will use them.

2 · 3 = 6

2 + 3 = 5

So, $x^2+5x+6=(x+2)(x+3)$

Answer: $\mathbf{(x+2)(x+3)}$

Check by multiplying: $(x+2)(x+3)=x^2+5x+6$

FACTORING

FACTORING OUT GCF AND FACTORING TRINOMIALS OF THE FORM: $\mathbf{x^2+bx+c}$

Problems	Answers
Factor	
1. $12x^5-15x^3+18x$	$3x\left(4x^4-5x^2+6\right)$
2. $21x^4y^5+14x^3y^3-7x^2y$	$7x^2y\left(3x^2y^4+2xy^2-1\right)$
3. $x^2-2x-63$	$\left(x-9\right)\left(x+7\right)$
4. $2x^2-20x+48$	$2\left(x-6\right)\left(x-4\right)$

FACTORING

FACTORING TRINOMIALS OF THE FORM: $ax^2 + bx + c$

Example: Factor $6x^2 + 14x + 4$

STEP			NOTES
1.	Factor out the GCF.	$6x^2 + 14x + 4$ $2(3x^2 + 7x + 2)$	GCF is 2.
2.	Identify the *a*, *b*, and *c*	$a = 3$; $b = 7$; and $c = 2$	for the expression $3x^2 + 7x + 2$
3.	Find the product of *ac*	$ac = 2 \bullet 3 = 6$	
4.	List all possible factor pairs that equal *ac* and identify the pair whose sum is *b*.	$\boxed{1 \cdot 6 \qquad 1 + 6 = 7}$ $2 \cdot 3 \qquad 2 + 3 = 5$	1 and 6 and 2 and 3 are the only factor pairs, however 1 and 6 is the correct pair.
5.	Replace the *bx* term with the boxed numbers.	$3x^2 \boxed{+7x} + 2$ $3x^2 \boxed{+1x + 6x} + 2$	You have replaced $+7x$ with $+1x + 6x$.
6.	Group the two pairs.	$(3x^2 + 1x) + (6x + 2)$	
7.	Factor the GCF out of each pair.	$x(3x + 1) + 2(3x + 1)$	The resulting common factor is $(3x + 1)$.
8.	Factor out $(3x + 1)$	$(3x + 1)(x + 2)$	These are the two factors of $3x^2 + 7x + 2$.
9.	Recall the original GCF from Step 1.	$2(2x^2 + 7x + 2) =$ $2(3x + 1)(x + 2)$	There are 3 factors of $6x^2 + 14x + 4$.
	Answer:	$\mathbf{2(3x + 1)(x + 2)}$	
10.	Check by multiplying.	$2(3x + 1)(x + 2) = 6x^2 + 14x + 4$	

FACTORING

FACTORING TRINOMIALS OF THE FORM: $ax^2 + bx + c$

Example: Factor $3a^2 - 4a - 4$

Hint: Recall the rule of signs: When *ac* is negative, the factor pairs have opposite signs.

STEP			NOTES
1.	Factor out the GCF.		None
2.	Identify the *a*, *b*, and *c*	$a = 3;\ b = -4;\ c = -4$	
3.	Find the product of *ac*	$ac = 3 \cdot (-4) = -12$	
4.	List all possible factor pairs that equal *ac* and identify the pair whose sum is *b*.	$+1 \cdot (-12) \quad +1 + (-12) = -11$ $-1 \cdot (+12) \quad -1 + (+12) = +11$ $\boxed{+2 \cdot (-6) \quad +2 + (-6) = -4}$ $-2 \cdot (+6) = -12 \quad -2 + (+6) = +4$ $+3 \cdot (-4) \quad +3 + (-4) = -1$ $-3 \cdot (+4) \quad -3 + (+4) = +1$	Include all possible factor combinations, including the +/– combinations since *ac* is negative.
5.	Replace the *bx* term with the boxed numbers.	$3a^2 - 4a - 4 = 3a^2 \underline{+2a - 6a} - 4$	Note: $-4a = 2a - 6a$
6.	Group the two pairs.	$= \underline{3a^2 + 2a}\ \ \underline{-6a - 4}$	Group by pairs. The resulting common factor is $(3a + 2)$
7.	Factor the GCF out of each pair.	$= a(3a + 2) - 2(3a + 2)$	
8.	Factor out $(3a + 2)$	$= (3a + 2)(a - 2)$	These are the factors.
	Answer: $\mathbf{(3a + 2)(a - 2)}$		
9.	Check to verify.	$(3a + 2)(a - 2) = 3a^2 - 4a - 4$	

FACTORING

FACTORING TRINOMIAL PROBLEMS OF THE FORM: ax^2+bx+c

Problems	Answers
Factor	
1. $6a^2-5a-6$	$(3a+2)(2a-3)$
2. $30a^2+25a-30$	$5(3a-2)(2a+3)$
3. $10x^2-27x+18$	$(2x-3)(5x-6)$
4. $20x^2+47x+21$	$(4x+7)(5x+3)$

FACTORING

FACTORING PERFECT SQUARE TRINOMIALS

Example: Factor $x^2+18x+81$

STEP		NOTES
1. Is there a GCF?		No
2. Determine if the trinomial is a Perfect Square.		
• Is the first term a perfect square?	x^2	Yes $x\cdot x=(x)^2$
• Is the third term a perfect square?	+81	Yes $9\cdot 9=(9)^2=81$
• Is the second term twice the product of the square roots of the first and third terms?	$+18x$	Yes $2(x\cdot 9)=18x$
3. To factor a Perfect Square Trinomial:		
• Find the square root of the first term.	x	$\sqrt{x^2}=x$
• Identify the sign of the second term	+	
• Find the square root of the third term.	9	$\sqrt{81}=9$
• Write the factored form.	$x^2+18x+81=(x+9)(x+9)=\boxed{(x+9)^2}$	
Answer: $(\mathbf{x+9})^2$		
• Check your work.	$(x+9)\cdot(x+9)=x^2+18x+81$	

Example: Factor $25x^2+20xy+4y^2$

STEP		NOTES
1. Is there a GCF?		No
2. Is the trinomial a Perfect Square Trinomial?		
• Is the first term a perfect square?	$25x^2$	Yes, $5x\cdot 5x=(5x)^2=25x^2$
• Is the third term a perfect square?	$4y^2$	Yes, $2y\cdot 2y=(2y)^2=4y^2$
• Is the second term twice the product of the square roots of the first and third terms?	$20xy$	Yes, $2(5x\cdot 2y)=20xy$
3. Factor the trinomial.		
• Square root of the first term	$\sqrt{25x^2}=5x$	
• Sign of the second term	$+20xy\rightarrow +$	
• Square root of the third term	$\sqrt{4y^2}=2y$	
• Factored form. **Answer**: $\mathbf{(5x+2y)^2}$	$25x^2+20xy+4y^2=(5x+2y)(5x+2y)=\boxed{(5x+2y)^2}$	
• Check your work.	$(5x+2y)\cdot(5x+2y)=25x^2+20xy+4y^2$	

FACTORING

FACTORING PERFECT SQUARE TRINOMIALS

Problems	Answers
Factor	
1. $16x^2 + 24x + 9$	$(4x+3)^2$
2. $4y^2 - 28y + 49$	$(2y-7)^2$
3. $9x^2 - 30xy + 25y^2$	$(3x-5y)^2$
4. $9x^2 + 30xy + 25y^2$	$(3x+5y)^2$

FACTORING

FACTORING THE DIFFERENCE OF TWO SQUARES

Example: Factor: $x^2 - 36$

STEP		NOTES
1. Is there a GCF?		No
2. Determine if the binomial is the Difference of Two Squares.		
• Can the binomial be written as a difference?		$x^2 \boxed{-} 36$, yes
• Is the first term a perfect square?	x^2	Yes, $x \cdot x = x^2$
• Is the second term a perfect square?	36	Yes, $6 \cdot 6 = 36$
3. To factor the Difference of Two Squares		
• Find the square root of the first term	x	$\sqrt{x^2} = x$
• Find the square root of the second term	6	$\sqrt{36} = 6$
• Write the factored form. **Answer**: $(x+6)(x-6)$	$x^2 - 36 = \boxed{(x+6)(x-6)}$	The factors are the product of the sum (+) and difference (–) of the terms' square roots.
• Check your work.	$(x+6)(x-6) = x^2 - 36$	

Example: Factor $36a^2 - 121b^2$

STEP		NOTES
1. Is there a GCF?		No
2. Is the binomial the Difference of Two Squares?		
• Can the binomial be written as a difference?	$36a^2 \boxed{-} 121b^2$	Yes
• Is the first term a perfect square?	$36a^2$	Yes, $6a \cdot 6a = 36a^2$
• Is the second term a perfect square?	$121b^2$	Yes, $11b \cdot 11b = 121b^2$
3. To factor the Difference of Two Squares		
• Find the square root of the first term	$6a$	$\sqrt{36a^2} = 6a$
• Find the square root of the second term	$11b$	$\sqrt{121b^2} = 11b$
• Write the factored form. **Answer**: $(6a+11b)(6a-11b)$	$36a^2 - 121b^2 =$ $\boxed{(6a+11b)(6a-11b)}$	The factors are the sum (+) and difference (–) of the terms' square roots.
• Check your work.	$(6a+11b)(6a-11b) = 36a^2 - 121b^2$	

FACTORING

FACTORING DIFFERENCE OF TWO SQUARES

Problems	Answers
Factor	
1. $9y^2-49$	$(3y+7)(3y-7)$
2. $25x^2-16$	$(5x-4)(5x+4)$
3. $4x^2-9y^2$	$(2x-3y)(2x+3y)$
4. $81a^4-16$	$(9a^2+4)(3a+2)(3a-2)$

FACTORING

FACTORING THE SUM OF TWO CUBES

Example: Factor $16x^5+250x^2y^3$

STEP			NOTES
1.	Is there a GCF?	$16x^5+250x^2y^3=2x^2\left(8x^3+125y^3\right)$	Yes, $2x^2$ is the GCF for both terms.
2.	Determine if the binomial is the Sum or Difference of Two Cubes.	$8x^3\boxed{+}125y^3$	Remember you are only looking at $8x^3+125y^3$; Sum.
•	Is the first term a perfect cube?	$8x^3$	Yes, $2x\cdot 2x\cdot 2x=\left(2x\right)^3=8x^3$
•	Is the second term a perfect cube?	$+125y^3$	Yes, $5y\cdot 5y\cdot 5y=\left(5y\right)^3=125y^3$
3.	To factor the Sum of Two Cubes, use $a^3x^3+b^3y^3=\left(ax+by\right)\left(a^2x^2-abxy+b^2y^2\right)$		where *a* and *b* are constants.
	To factor $8x^3+125y^3$:		
•	Find the cube root of the first term.	$2x$	$\sqrt[3]{8x^3}=2x$
•	Find the cube root of the second term.	$5y$	$\sqrt[3]{125y^3}=5y$
•	Using the formula above, write the factored form.	$8x^3+125y^3=\left(2x+5y\right)\left(4x^2-10xy+25y^2\right)$	Note the sign of the $10xy$ is opposite that of the $+125y^3$.

Recall, you factored out the GCF $2x^2$ from the original binomial. You must now include it in the final factored form.

$$16x^5+250x^2y^3=\boxed{2x^2\left(2x+5y\right)\left(4x^2-10xy+25y^2\right)}$$

Answer: $\mathbf{2x^2\left(2x+5y\right)\left(4x^2-10xy+25y^2\right)}$

FACTORING

FACTORING THE DIFFERENCE OF TWO CUBES

Example: Factor $64x^3-27y^3$

STEP		NOTES
1. Is there a GCF?	$64x^3-27y^3$	No
2. Determine if the binomial is the Sum or Difference of Two Cubes.	$64x^3\boxed{-}27y^3$	Difference
• Is the first term a perfect cube?	$64x^3$	Yes, $4x\cdot 4x\cdot 4x=(4x)^3=64x^3$
• Is the second term a perfect cube?	$27y^3$	Yes, $3y\cdot 3y\cdot 3y=(3y)^3=27y^3$
3. To factor the Difference of Two Cubes, use $a^3x^3-b^3y^3=(ax-by)(a^2x^2+abxy+b^2y^2)$		Where *a* and *b* are constants.
To factor $64x^3-27y^3$:		
• Find the cube root of the first term.	$4x$	$\sqrt[3]{64x^3}=4x$
• Find the cube root of the second term.	$3y$	$\sqrt[3]{27y^3}=3y$
• Using the formula above, write the factored form.	$64x^3-27y^3=\boxed{(4x-3y)(16x^2+12xy+9y^2)}$	Note the sign of 12*xy* is opposite that of the $-27y^3$.

FACTORING

FACTORING THE SUM and DIFFERENCE OF TWO CUBES

Problems	Answers
Factor	
1. $27x^3+8$	$(3x+2)(9x^2-6x+4)$
2. $64a^3-125b^3$	$(4a-5b)(16a^2+20ab+25b^2)$
3. $24y^5-3y^2$	$3y^2(2y-1)(4y^2+2y+1)$

FACTORING

FACTORING STRATEGY

1. Is there a common factor? If so, factor out the GCF, or the opposite of the GCF so that the leading coefficient is positive.

2. How many terms does the polynomial have?

   If it has *two terms*, look for the following problem types:

   a. The difference of two squares
   b. The sum of two cubes
   c. The difference of two cubes

   If it has *three terms*, look for the following problem types:

   a. A perfect-square trinomial
   b. If the trinomial is not a perfect square, use the grouping method.

   If it has *four or more terms*, try to factor by grouping.

3. Can any factors be factored further? If so, factor them completely.

4. Does the factorization check? Check by multiplying.

FACTORING STRATEGY

Always check for

Greatest Common Factor first!

Then continue:

2 – Terms

Difference of Two Squares

$$x^2-4$$

$$(x)^2-(2)^2$$

$$(x+2)(x-2)$$

Sum and Difference of Two Cubes

$$x^3-8=(x-2)(x^2+2x+4)$$

$$x^3+8=(x+2)(x^2-2x+4)$$

3 – Terms

Form: x^2+bx+c

$$x^2-6x+5$$

List factors of 5

$$5\cdot 1$$

$$\boxed{-5\cdot -1}$$

Choose the pair which adds to –6

$$-5+-1=-6$$

$$x^2-5x-1x+5$$

$$x(x-5)-1(x-5)$$

$$(x-5)(x-1)$$

Perfect Square Trinomials

$$4x^2+20x+25$$

$$(2x)^2+2(2x)(5)+(5)^2$$

$$(2x+5)(2x+5)$$

$$(2x+5)^2$$

Form: ax^2+bx+c

$$5x^2-7x-6$$

$$5\cdot -6=-30$$

List factors of –30

$-1\cdot 30$	$1\cdot -30$
$-2\cdot 15$	$2\cdot -15$
$-3\cdot 10$	$\boxed{3\cdot -10}$
$-5\cdot 6$	$5\cdot -6$

Choose the pair which add to –7

$$3+-10=-7$$

Substitute this pair in for the middle term.

$$5x^2+3x-10x-6$$

Factor out the GCF of each pair of terms

$$x(5x+3)-2(5x+3)$$

$$(5x+3)(x-2)$$

4 – Terms

Factor by Grouping

$$7x^2+14x-6x-12$$

$$7x(x+2)-6(x+2)$$

$$(x+2)(7x-6)$$

FACTORING

Problems

Factor.

1. $x^3+4x^2-9x-36$
2. $8x^3+12x^2-2x-3$
3. $9x^3-18x^2-4x+8$
4. $12x^3+8x^2-27x-18$
5. $32x^3-80x^2-18x+45$
6. $3x^3-4x^2-75x+100$
7. $8x^2-14x-15$
8. $9x^2+6x-8$
9. $3x^2-13x+12$
10. $6x^2+19x-20$
11. $4x^2+7x-2$
12. $4x^2+5x-6$
13. $16x^2-36$
14. $27x^4-48x^2$
15. $100x^2-16$
16. $16x^4-81$
17. $27x^3+8$
18. $1-64a^3$
19. $125p^3-64q^3$
20. $80x^3-10$
21. $16x^3+250$
22. x^4-27x

FACTORING

Answers

1. $(x+3)(x-3)(x+4)$
2. $(2x+1)(2x-1)(2x+3)$
3. $(3x+2)(3x-2)(x-2)$
4. $(2x+3)(2x-3)(3x+2)$
5. $(4x+3)(4x-3)(2x-5)$
6. $(x+5)(x-5)(3x-4)$
7. $(4x+3)(2x-5)$
8. $(3x-2)(3x+4)$
9. $(3x-4)(x-3)$
10. $(6x-5)(x+4)$
11. $(4x-1)(x+2)$
12. $(4x-3)(x+2)$
13. $4(2x+3)(2x-3)$
14. $3x^2(3x+4)(3x-4)$
15. $4(5x+2)(5x-2)$
16. $(4x^2+9)(2x+3)(2x-3)$
17. $(3x+2)(9x^2-6x+4)$
18. $(1-4a)(1+4a+16a^2)$
19. $(5p-4q)(25p^2+20pq+16q^2)$
20. $10(2x-1)(4x^2+2x+1)$
21. $2(2x+5)(4x^2-10x+25)$
22. $x(x-3)(x^2+3x+9)$

QUADRATIC EQUATIONS

SOLVING QUADRATIC EQUATIONS BY FACTORING

1. Write the equation in standard form: $ax^2+bx+c=0$ or $0=ax^2+bx+c$.
2. Factor completely.
3. Use the zero-factor property to set each factor equal to 0.
4. Solve each resulting linear equation.
5. Check the results in the original equation.

Example: Solve $x^2-100=0$

This equation is in standard from.

$$(x-10)(x+10)=0$$

Factor completely.

$$x-10=0 \qquad x+10=0$$

Set each factor equal to 0.

$$x=10 \qquad x=-10$$

Solve each linear equation.

Check: $x=10$

$$\begin{aligned} x^2-100&=0\\ (10)^2-100&=0\\ 100-100&=0\\ 0&=0 \text{ True} \end{aligned}$$

Check: $x=-10$

$$\begin{aligned} x^2-100&=0\\ (-10)^2-100&=0\\ 100-100&=0\\ 0&=0 \text{ True} \end{aligned}$$

The solutions of $x^2-100=0$ are $x=10$ and $x=-10$.

Example: Solve $6x^2=7x+3$

$$6x^2-7x-3=0$$

This equation is in standard form.

$$(3x+1)(2x-3)=0$$

Factor completely.

$$3x+1=0 \qquad 2x-3=0$$

Set each factor equal to 0.

$$x=-\frac{1}{3} \qquad x=\frac{3}{2}$$

Solve each linear equation.

Check: $x=-\frac{1}{3}$

$$\begin{aligned} 6x^2&=7x+3\\ 6\left(-\frac{1}{3}\right)^2&=7\left(-\frac{1}{3}\right)+3\\ 6\cdot\frac{1}{9}&=-\frac{7}{3}+\frac{9}{3}\\ \frac{2}{3}&=\frac{2}{3} \text{ True} \end{aligned}$$

Check: $x=\frac{3}{2}$

$$\begin{aligned} 6x^2&=7x+3\\ 6\left(\frac{3}{2}\right)^2&=7\left(\frac{3}{2}\right)+3\\ {}^{3}\cancel{6}\cdot\frac{9}{\cancel{4}_{2}}&=\frac{21}{2}+\frac{6}{2}\\ \frac{27}{2}&=\frac{27}{2} \text{ True} \end{aligned}$$

The solutions of $6x^2=7x+3$ are $x=-\frac{1}{3}$ and $x=\frac{3}{2}$.

QUADRATIC EQUATIONS

Problems	Answers
Solve	
1. $x^2+3x-70=0$	$x=7,\ x=-10$
2. $2x^2+11x=-15$	$x=-\frac{5}{2}\ \ x=-3$
3. $25=4t^2$	$t=\frac{5}{2}\quad t=-\frac{5}{2}$
4. $-18y^2-6y=0$	$y=0\ \ y=-\frac{1}{3}$
5. $x\left(3x+10\right)=-8$	$x=-\frac{4}{3}\ \ x=-2$
6. $25x^2+9=30x$	$x=\frac{3}{5}$
7. $x^3=x\left(x+6\right)$	$x=0,\ \ x=3,$ $x=-2$

APPLICATION OF QUADRATIC EQUATIONS

Example: Sharon and Diane went to the deli department at a local grocery store. Sharon arrived first and took a number to reserve her turn for service. Diane took the next number. If the product of their two numbers was 110, what were their numbers?

Define the variable. n = Sharon’s number

$n + 1$ = Diane’s number

Write and equation $n\left(n+1\right)=110$

Solve the equation

$$\begin{cases} n^2+n=110 \\ n^2+n-110=0 \\ \left(n+11\right)\left(n-10\right)=0 \\ \cancel{n=-11} \qquad n=10 \end{cases}$$

-11 is an extraneous solution

Sharon’s number: $n=10$, Diane’s number: $n+1=11$

Check in the original problem.

The answers are: Sharon took number 10 and Diane took number 11.

Example: Anita’s hat blows off her head at the top of an amusement park ride. The height *h*, in feet, of the hat *t* seconds after it blew off her head is given by $h=-16t^2+16t+32$. After how many seconds will the hat hit the ground?

Define the variables h=height of the hat (in feet)

t =time (in seconds)

Use the given equation $h=-16t^2+16t+32$.

Substitute $h = 0$ because the height of the hat is 0 feet when it hits the ground.

Solve the Equation

$$\begin{cases} 0=-16t^2+16t+32 \\ 0=-16\left(t^2-t-2\right) \\ 0=-16\left(t-2\right)\left(t+1\right) \\ t-2=0 \qquad \cancel{t+1=0} \\ t=2 \qquad \cancel{t=-1} \end{cases}$$

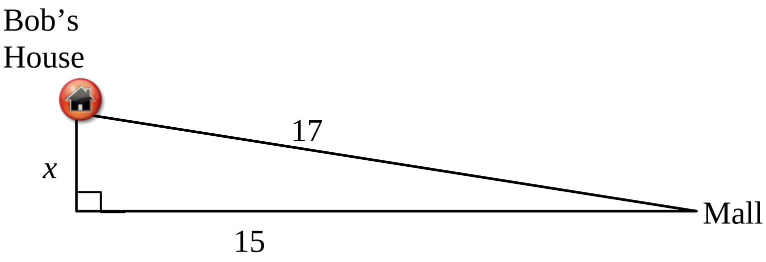
–1 is an extraneous solution

Check in the original problem.

The answer is: It took 2 seconds for the hat to hit the ground.

APPLICATION OF QUADRATIC EQUATIONS

Solve

Problems	Answers
1. Bob has two routes he could take to get from his house to the mall.  One route is direct and along a country road for 17 miles. The other route is x miles south and 15 miles due east to the mall. a) Find the distance x. b) Determine how many miles he saves driving the direct route.	a) 8 miles b) 6 miles
2. Phil teaches in two different rooms. The room numbers are consecutive even integers whose product is 168. Find the two room numbers.	12 and 14
3. The area of a garden is $495\,\text{ft.}^2$ The length is 3 feet more than twice the width. Find the dimensions of the garden.	15 ft. and 33 ft.
4. Suppose that a cannon ball is shot upward with an initial speed of 80 ft./second. The height h in feet of the cannon ball t seconds after being shot is given by $h = -16t^2 + 80t$. After how many seconds will it hit the ground?	5 seconds

RATIONAL EXPRESSIONS

SIMPLIFYING RATIONAL EXPRESSIONS

Example: Simplify $\frac{x^2-x-6}{x^2-3x}$

To simplify, factor numerator and denominator completely and cancel common factors.

Recall $\frac{x-3}{x-3}=1$.

$$\frac{x^2-x-6}{x^2-3x}=\frac{(x+2)\overset{1}{\cancel{(x-3)}}}{x\underset{1}{\cancel{(x-3)}}}=\frac{x+2}{x}$$

Example: Simplify $\frac{-x^2+3x+4}{x^2+6x+5}$

Recall: Step 1 of factoring strategy: The leading coefficient must be positive. Continue by factoring the numerator and denominator completely. Cancel common factors.

$$\begin{aligned}\frac{-x^2+3x+4}{x^2+6x+5}&=\frac{-1\left(x^2-3x-4\right)}{x^2+6x+5}\\&=\frac{-1(x-4)\overset{1}{\cancel{(x+1)}}}{\underset{1}{\cancel{(x+1)}}(x+5)}\\&=\frac{-(x-4)}{x+5}\end{aligned}$$

OPERATIONS WITH RATIONAL EXPRESSIONS

To add, subtract, multiply or divide rational expressions, use the following rules for adding, subtracting, multiplying or dividing fractions.

Recall:

1. Multiply two fractions by multiplying the numerators and multiplying the denominators. Use cancellation, if necessary, to write the product in lowest terms.

Example: Simplify $\frac{x^2-9}{x+3}\bullet\frac{2x+6}{4}$

$$\frac{x^2-9}{x+3}\cdot\frac{2x+6}{4}=\frac{(x+3)(x-3)}{\underset{1}{\cancel{x+3}}}\cdot\frac{\overset{1}{\cancel{2}}\overset{1}{\cancel{(x+3)}}}{\underset{2}{\cancel{4}}}=\frac{(x+3)(x-3)}{2}$$

RATIONAL EXPRESSIONS

2. Divide two fractions by inverting the second fraction (divisor) and multiplying.

Example: Simplify $\frac{a^2-1}{a-1}\div\frac{a^2-2a+1}{a+1}$

$$\frac{a^2-1}{a-1}\div\frac{a^2-2a+1}{a+1}=\frac{a^2-1}{a-1}\cdot\frac{a+1}{a^2-2a+1}$$

$$=\frac{(a+1)\overset{1}{\cancel{(a-1)}}}{(a-1)}\cdot\frac{a+1}{\underset{1}{\cancel{(a-1)}}(a-1)}$$

$$=\frac{(a+1)^2}{(a-1)^2}$$

3. To add (or subtract) like fractions, add the numerators only and place your result over the common denominator.

Example: Simplify $\frac{3}{x+2}+\frac{4x}{x+2}$

$$\frac{3}{x+2}+\frac{4x}{x+2}=\frac{3+4x}{x+2}=\frac{4x+3}{x+2}$$

4. To add (or subtract) unlike fractions, find the lowest common denominator, rewrite fractions with the common denominator, and add (or subtract) numerators, placing the answer over the common denominator.

Example: Simplify $\frac{4}{x^2-4}+\frac{2}{x+2}$

$$\frac{4}{x^2-4}+\frac{2}{x+2}=\frac{4}{(x+2)(x-2)}+\frac{2(x-2)}{(x+2)(x-2)}$$

Lowest common denominator is x^2-4 which is $(x+2)(x-2)$.

$$=\frac{4}{(x+2)(x-2)}+\frac{2x-4}{(x+2)(x-2)}$$

$$=\frac{4+2x-4}{(x+2)(x-2)}=\frac{2x}{(x+2)(x-2)}$$

IMPORTANT NOTE: All answers must be simplified, i.e. reduced to lowest terms.

RATIONAL EXPRESSIONS

Examples:

1. Multiply:

$$\begin{aligned}\frac{x+2}{x-3}\cdot\frac{x^2-4}{x^2+x-2} &= \frac{\cancel{x+2}}{x-3}\cdot\frac{(x+2)(x-2)}{(\cancel{x+2})(x-1)}\\ &= \frac{(x+2)(x-2)}{(x-3)(x-1)}\end{aligned}$$

2. Multiply:

$$\begin{aligned}\frac{1-a^3}{a^2}\cdot\frac{a^5}{a^2-1} &= \frac{-1\left(a^3-1\right)}{a^2}\cdot\frac{a^5}{a^2-1}\\ &= \frac{-1\overset{1}{\cancel{(a-1)}}\left(a^2+a+1\right)}{\underset{1}{\cancel{a^2}}}\cdot\frac{\overset{a^3}{\cancel{a^5}}}{(a+1)\underset{1}{\cancel{(a-1)}}}\\ &= \frac{-1\left(a^2+a+1\right)a^3}{a+1}\\ &= \frac{-a^3\left(a^2+a+1\right)}{a+1}\end{aligned}$$

3. Divide:

$$\begin{aligned}\frac{3x^2-x-2}{x^2-x}\div\frac{3x^2+14x+8}{x^2-3x-28} &= \frac{3x^2-x-2}{x^2-x}\cdot\frac{x^2-3x-28}{3x^2+14x+8}\\ &= \frac{\overset{1}{\cancel{(3x+2)}}\overset{1}{\cancel{(x-1)}}}{x\underset{1}{\cancel{(x-1)}}}\cdot\frac{\overset{1}{\cancel{(x+4)}}(x-7)}{\underset{1}{\cancel{(3x+2)}}\underset{1}{\cancel{(x+4)}}}\\ &= \frac{x-7}{x}\end{aligned}$$

4. Subtract like fractions:

$$\begin{aligned}\frac{4x+5}{x+3}-\frac{x-4}{x+3} &= \frac{4x+5-(x-4)}{(x+3)}\\ &= \frac{4x+5-x+4}{(x+3)}\\ &= \frac{3x+9}{x+3}\\ &= \frac{3\overset{1}{\cancel{(x+3)}}}{\underset{1}{\cancel{(x+3)}}}\\ &= 3\end{aligned}$$

RATIONAL EXPRESSIONS

5. Subtract unlike fractions:

$$\frac{5x}{x-2y}-\frac{3y-7}{2y-x}=\frac{5x}{x-2y}-\frac{3y-7}{-1(x-2y)}$$

$$=\frac{5x}{x-2y}+\frac{3y-7}{x-2y}$$

$$=\frac{5x+3y-7}{x-2y}$$

6. Subtract unlike fractions:

$$\frac{2y+1}{y^2-7y+6}-\frac{y+3}{y^2-5y-6}=\frac{2y+1}{(y-6)(y-1)}-\frac{y+3}{(y-6)(y+1)}$$

$$=\frac{(2y+1)(y+1)}{(y-6)(y-1)(y+1)}-\frac{(y+3)(y-1)}{(y-6)(y+1)(y-1)}$$

$$=\frac{(2y+1)(y+1)-(y+3)(y-1)}{(y-6)(y-1)(y+1)}$$

$$=\frac{2y^2+3y+1-(y^2+2y-3)}{(y-6)(y-1)(y+1)}$$

$$=\frac{2y^2+3y+1-y^2-2y+3}{(y-6)(y-1)(y+1)}=\frac{y^2+y+4}{(y-6)(y-1)(y+1)}$$

RATIONAL EXPRESSIONS

Problems	Answers
Perform any operations. Simplify.	
1. $\frac{4x^2-32x+60}{2x^2-3x-9}$	$\frac{4(x-5)}{2x+3}$
2. $\frac{x^2}{x+2}\cdot\frac{x^2+4x+4}{x}$	$x(x+2)$
3. $\frac{2x-12}{20x}\cdot\frac{8x^3}{3x-18}$	$\frac{4x^2}{15}$
4. $\frac{x+y}{x-y}\div\frac{3x+3y}{4x-4y}$	$\frac{4}{3}$
5. $\frac{2x^2+3x-2}{x^2-x-20}\cdot\frac{x^2+x-12}{x^2+3x+2}$	$\frac{(2x-1)(x-3)}{(x-5)(x+1)}$
6. $\frac{x^2+7x+12}{x-5}\cdot\frac{x^2-7x+10}{x^2+6x+8}$	$\frac{(x+3)(x-2)}{x+2}$
7. $\frac{x^2+4x+4}{x^2-4x+4}\div\frac{(x+2)^2}{(x-2)^2}$	1
8. $\frac{x-3}{x^2+2x-3}\cdot\frac{x^2-2x+1}{x^2-2x-3}\div\frac{x^2-9}{x^2-1}$	$\frac{(x-1)^2}{(x+3)^2(x-3)}$
9. $\frac{x}{x+2}-\frac{4}{x+2}$	$\frac{x-4}{x+2}$

RATIONAL EXPRESSIONS

Problems	Answers
10. $\frac{3x}{x+3}+\frac{9}{x+3}$	3
11. $\frac{4}{x+2}+\frac{x}{2x+4}$	$\frac{8+x}{2(x+2)}$
12. $\frac{3}{x+2}+\frac{4}{x-1}$	$\frac{7x+5}{(x+2)(x-1)}$
13. $\frac{2}{x-3}-\frac{4}{x-1}$	$\frac{-2x+10}{(x-3)(x-1)}$
14. $\frac{5}{x^2-4}+\frac{4}{x+2}$	$\frac{4x-3}{(x+2)(x-2)}$
15. $\frac{3}{x-1}-\frac{8}{1-x}$	$\frac{11}{x-1}$
16. $\frac{10}{3x+3y}+\frac{13}{2x+2y}$	$\frac{59}{6(x+y)}$
17. $\frac{x}{x-3}+\frac{4}{x^2-x-6}$	$\frac{x^2+2x+4}{(x-3)(x+2)}$

Note: To simplify rational expressions, factor the numerator and the denominator completely.
Then cancel common factors to write the rational expression in lowest terms.

COMPLEX FRACTIONS

SIMPLIFYING COMPLEX FRACTIONS

1. If needed, add or subtract in the numerator and/or denominator so that the numerator is a single fraction and the denominator is a single fraction.
2. Perform the indicated division by multiplying the numerator of the complex fraction by the reciprocal of the denominator.
3. Simplify the result, if possible.

Example: Simplify $\dfrac{\frac{2}{3}+\frac{1}{x}}{\frac{x}{3}-\frac{1}{4}}$

$$\frac{\frac{2}{3}+\frac{1}{x}}{\frac{x}{3}-\frac{1}{4}} = \frac{\frac{2}{3}\cdot\frac{x}{x}+\frac{1}{x}\cdot\frac{3}{3}}{\frac{x}{3}\cdot\frac{4}{4}-\frac{1}{4}\cdot\frac{3}{3}}$$

The LCD of the numerator of the complex fraction is 3*x*.
The LCD of the denominator of the complex fraction is 12.

$$= \frac{\frac{2x}{3x}+\frac{3}{3x}}{\frac{4x}{12}-\frac{3}{12}}$$

$$= \frac{\frac{2x+3}{3x}}{\frac{4x-3}{12}}$$

Simplify the numerator.

Simplify the denominator.

$$= \frac{2x+3}{3x}\div\frac{4x-3}{12}$$

$$= \frac{2x+3}{\underset{1}{\cancel{3}}x}\cdot\frac{\overset{4}{\cancel{12}}}{4x-3}$$

Change division to multiplication by the reciprocal.

Cancel common factors.

$$= \frac{4(2x+3)}{x(4x-3)}$$

COMPLEX FRACTIONS

Problems	Answers
Simplify.	
1. $\dfrac{\frac{4}{9}-\frac{5}{y}}{\frac{y}{2}+\frac{1}{6}}$	$\dfrac{2(4y-45)}{3y(3y+1)}$
2. $\dfrac{\frac{5x}{y}-x}{x-\frac{2x}{y}}$	$\dfrac{5-y}{y-2}$ or $\dfrac{-y+5}{y-2}$
3. $\dfrac{\frac{8}{b^2}}{\frac{2}{5}+\frac{4}{b}}$	$\dfrac{20}{b(b+10)}$

RATIONAL EQUATIONS

STEPS TO SOLVE RATIONAL EQUATIONS

1. Determine which numbers cannot be solutions of the equation.
2. Multiply all terms in the equation by the LCD of all rational expressions in the equation. This clears the equation of fractions.
3. Solve the resulting equation.
4. Check all possible solutions in the original equation.

Example: Solve $\frac{2}{y-1}+\frac{y-2}{3}=\frac{4}{y-1}$

STEP		NOTES
1.	$y-1\neq 0$ $y\neq 1$	The denominator equals 0 when $y=1$. Therefore, $y=1$ cannot be a solution.
2.	$3\overset{1}{\cancel{(y-1)}}\left(\frac{2}{\underset{1}{\cancel{y-1}}}\right)+\overset{1}{\cancel{3}}(y-1)\left(\frac{y-2}{\underset{1}{\cancel{3}}}\right)=3\overset{1}{\cancel{(y-1)}}\left(\frac{4}{\underset{1}{\cancel{y-1}}}\right)$	Multiply all terms in the equation by the LCD of $3(y-1)$. Cancel common factors.
3.	$3(2)+(y-1)(y-2)=3(4)$	This clears the equation of fractions.
	$\left.\begin{aligned}6+y^2-3y+2&=12\\ y^2-3y-4&=0\\ (y-4)(y+1)&=0\\ y-4=0 \quad & y+1=0\\ y=4 \quad & y=-1\end{aligned}\right\}$	Solve the equation
4.	Check $y=4$ and $y=-1$	Check all possible solutions in the original equation.

Check $y=4$

$$\frac{2}{4-1}+\frac{4-2}{3}=\frac{4}{4-1}$$
$$\frac{2}{3}+\frac{2}{3}=\frac{4}{3}$$
$$\checkmark\ \frac{4}{3}=\frac{4}{3}$$

Check $y=-1$

$$\frac{2}{-1-1}+\frac{-1-2}{3}=\frac{4}{-1-1}$$
$$\frac{2}{-2}+\frac{-3}{3}=\frac{4}{-2}$$
$$-1+-1=-2$$
$$\checkmark\ -2=-2$$

The solutions are $y=4$ and $y=-1$.

RATIONAL EQUATIONS

Example: Solve $2+\frac{5}{y-4}=\frac{y+1}{y-4}$

	STEP	NOTES
1.	$y-4\neq 0$ $y\neq 4$	The denominator equals 0 when *y* = 4. Therefore, *y* = 4 cannot be a solution.
2.	$(y-4)(2)+\overset{1}{\cancel{(y-4)}}\left(\frac{5}{\underset{1}{\cancel{y-4}}}\right)=\underset{1}{\cancel{(y-4)}}\left(\frac{y+1}{\underset{1}{\cancel{y-4}}}\right)$	Multiply all terms in the equation by the LCD of y − 4. Cancel common factors.
3.	$(y-4)(2)+5=y+1$	This clears the equation of fractions.
	$\left.\begin{aligned}&2y-8+5=y+1\\&2y-3=y+1\\&y=4\end{aligned}\right\}$	Solve the equation.
4.	Check *y* = 4. $2+\frac{5}{4-4}=\frac{4+1}{4-4}$ $2+\frac{5}{0}=\frac{5}{0}$	Check all possible solutions in the original equation. Cannot divide by 0. See Step 1. *y* = 4 cannot be a solution.

No solution (4 is extraneous)

RATIONAL EQUATIONS

Example: Solve $\dfrac{x}{x+1}=\dfrac{6}{x+7}$

STEP	NOTES
1. $x+1\neq 0$ $\quad$ $x+7\neq 0$ $x\neq -1$ $\quad$ $x\neq -7$	The denominator equals 0 when $x=-1$ and $x=-7$. Therefore $x=-1$ and $x=-7$ cannot be solutions.
2. $x\left(x+7\right)=6\left(x+1\right)$	This example is a proportion so we can use the cross product.
3. $\left.\begin{aligned} x^2+7x&=6x+6\\ x^2+x-6&=0\\ \left(x+3\right)\left(x-2\right)&=0\\ x=-3 \quad x&=2 \end{aligned}\right\}$	Solve this equation.
4. Remember to Check.	

The solutions are $x=-3$ and $x=2$.

RATIONAL EQUATIONS

Problems	Answers
Solve	
1. $\frac{1}{y-1}=1-\frac{3}{y-1}$	$y=5$
2. $\frac{x+2}{x+8}=\frac{x-3}{x-2}$	$x=4$
3. $\frac{y}{y+1}=\frac{6}{y+7}$	$y=-3, \quad y=2$
4. $\frac{y}{y-5}-3=\frac{5}{y-5}$	No solution (5 is extraneous)
5. $\frac{x}{x^2-9}=\frac{x-8}{x-3}-\frac{x+8}{x+3}$	$x=0$

APPLICATIONS OF RATIONAL EQUATIONS

NUMBER PROBLEM

Example: If the same number is added to both the numerator and denominator of the fraction $\frac{4}{7}$, the result is $\frac{3}{4}$. Find the number.

Let x = the unknown number

Define the variable.

$$\frac{4+x}{7+x}=\frac{3}{4}$$

Form an equation.

$$\left.\begin{aligned}\frac{4+x}{7+x}&=\frac{3}{4}\\ 4(4+x)&=3(7+x)\\ 16+4x&=21+3x\\ 16+x&=21\\ x&=5\end{aligned}\right\}$$

Solve the equation.

Check:

$$4+5=9$$
$$7+5=12$$
$$\frac{9}{12}=\frac{3}{4}$$
$$\frac{3}{4}=\frac{3}{4}$$

Check the result in the original word problem.

Answer: The number is 5.

State the conclusion.

APPLICATIONS OF RATIONAL EQUATIONS

UNIFORM MOTION PROBLEM

Distance = Rate • Time $\qquad d = r \cdot t \qquad r = \dfrac{d}{t} \qquad t = \dfrac{d}{r}$

Example: Tammi bicycles at a rate of 6 mph faster than she walks. In the same time it takes her to bicycle 5 miles she can walk 2 miles. How fast does she walk?

	Rate ·	Time =	Distance	Define the variable: $w =$ Tammi's walking rate
bicycling	$w+6$	$\dfrac{5}{w+6}$	5	
walking	w	$\dfrac{2}{w}$	2	

$$\frac{5}{w+6} = \frac{2}{w}$$

Form an equation which sets the times equal to each other. (bicycling time = walking time)

$$\left.\begin{aligned} 5w &= 2(w+6) \\ 5w &= 2w+12 \\ 3w &= 12 \\ w &= 4 \end{aligned}\right\}$$

Solve the equation.

Check:

walking rate is 4 mph $\quad (w = 4)$

bicycling rate is 10 mph $\quad (w + 6 = 10)$

Check the result in the original word problem.

Using $t = \dfrac{d}{r}$, show that the walking time equals the bicycling time

walking	bicycling
$t = \dfrac{2\,\text{miles}}{4\,\text{mph}}$	$t = \dfrac{5\,\text{miles}}{10\,\text{mph}}$
$t = \dfrac{1}{2}\,\text{hour}$	$t = \dfrac{1}{2}\,\text{hour}$

The times are the same.

Answer: **Tammi's walking rate is 4 mph.**

State the conclustion.

APPLICATIONS OF RATIONAL EQUATIONS

WORK SHARED PROBLEM

Work Completed = Work Rate • Time Worked $w = r \cdot t$

Work Completed by Unit 1 + Work Completed by Unit 2 = 1 Job Completed

Example: It takes Charlie 6 hours to prepare the soccer field for a game. It takes Scotty 8 hours to prepare the same field. How long will it take if they work together?

	Work rate $\cdot$ Time worked = Work completed			
Charlie	$\frac{1}{6}$	t	$\frac{t}{6}$	Define the variable: $t =$ time in hours that they worked together.
Scotty	$\frac{1}{8}$	t	$\frac{t}{8}$	

Work Completed by Charlie + Work Completed by Scotty = 1 Job Completed

$$\frac{t}{6} + \frac{t}{8} = 1$$

Form an equation.

$$\left.\begin{aligned} 24\cdot\frac{t}{6} + 24\cdot\frac{t}{8} &= 24\cdot 1 \\ 4t + 3t &= 24 \\ 7t &= 24 \\ t = \frac{24}{7} &= 3\frac{3}{7}\text{ hours} \end{aligned}\right\}$$

Solve the equation.

Check:

Using $w = r \cdot t$

Charlie's work completed	Scotty's work completed	Check the result in the original word problem.
$w = \frac{1}{\cancel{6}}\cdot\frac{\cancel{24}^{4}}{7}$	$w = \frac{1}{\cancel{8}}\cdot\frac{\cancel{24}^{3}}{7}$	
$w = \frac{4}{7}$ of the job	$w = \frac{3}{7}$ of the job	

$$\frac{4}{7} + \frac{3}{7} = 1 \text{ job completed}$$

Answer: **It took them $3\frac{3}{7}$ hrs. working together.**

State the conclusion.

APPLICATIONS OF RATIONAL EQUATIONS

SIMPLE INTEREST PROBLEM

Interest = Principal • Rate • Time $\quad I = p \cdot r \cdot t \quad \frac{I}{rt} = p \quad \frac{I}{pr} = t$

Example: An amount of money can be invested in a mutual fund which would yield $350 in one year. The same amount of money invested in a bond would yield only $150 in a year, because the interest paid is 2% less than that paid by the mutual fund. Find the rate of interest paid by each investment.

	Principal ·	Rate ·	Time =	Interest	
Mutual Fund	$\frac{350}{1 \cdot r}$	r	1	350	Define the variable: $r =$ rate of interest paid by mutual fund
Bond	$\frac{150}{1(r-.02)}$	$r-.02$	1	150	

$$\frac{350}{r} = \frac{150}{r-.02}$$

Form an equation which sets the principals equal to each other.

$$\left.\begin{aligned} 350r - 7 &= 150r \\ 200r &= 7 \\ r &= .035 = 3.5\% \\ r - .02 &= .015 = 1.5\% \end{aligned}\right\}$$

Solve the equation.

Check:

Find the principal for each investment using $P = \frac{I}{rt}$

Check the result in the original word problem.

Mutual Fund

$$P = \frac{350}{(.035)(1)} = \$10,000$$

Bond

$$P = \frac{150}{(.015)(1)} = \$10,000$$

The principals are the same.

Answer:

The interest rate for the mutual fund is 3.5%
The interest rate for the bond is 1.5%

State the conclusion

APPLICATIONS OF RATIONAL EQUATIONS

Problems	Answers
1. If the same number is subtracted from both the numerator and the denominator of $\frac{10}{19}$, the result is $\frac{2}{5}$. Find the number.	4
2. If the numerator of $\frac{5}{6}$ is increased by a certain number and the denominator is doubled, the result is $\frac{2}{3}$. Find the number.	3
3. Melissa can drive 110 miles in the same time as it takes her to bike 30 miles. If she can drive 40 mph faster than she can bike, how fast can she bike?	15 mph
4. Mike can bicycle 36 miles in the same time that he can jog 9 miles. If he can bike 9 mph faster than he can jog, how fast can he bike?	12 mph

APPLICATIONS OF RATIONAL EQUATIONS

Problems	Answers
5. In 1 year, an investor earned $60 interest on money he deposited in his savings account at a bank. He later learned that the money would have earned $150 had he invested in municipal bonds, because the bonds paid 3% more interest at the time. Find the rate he received in his savings account. Also, find the principal amount he put in the savings account.	2% and $3000
6. A plane flies 780 miles downwind in the same amount of time as it takes to travel 580 upwind. If the plane can fly 340 mph in still air, find the velocity of the wind.	50 mph
7. Two bond funds pay interest rates that differ by 1%. Money invested for 1 year in the first fund earns $240 interest. The same amount invested in the second fund earns $160. Find both the lower rate of interest and the principal amount of the investment.	2% and $8000
8. If the manager of a pet store takes 8 hours to clean all of the cages and it takes his assistant 2 hours more than that to clean the same cages, how long will it take if they work together?	$4\frac{4}{9}$ hours
9. If it takes a night security guard to make his rounds on campus 2 hours and it takes his assistant 3 hours, how long will it take them working together?	$1\frac{1}{5}$ hours

LINEAR EQUATIONS

The **standard form** of the equation of a line is written as

$$ax + by + c = 0$$

where *a* and *b* are not both 0.

Definition of slope: Let *L* be a line passing through the points $(x_1 y_1)$ and $(x_2 y_2)$. Then the slope of *L* is given by the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

The **slope-intercept form** of the equation of a line is written as

$$y = mx + b$$

where *m* is the slope and the point $(0, b)$ is the y-intercept.

The **point-slope form** of the equation of a line passing through (x_1, y_1) and having slope m is written as

$$y - y_1 = m(x - x_1)$$

where *m* is the slope and (x_1, y_1) is the point.

Parallel Property: Parallel lines have the same slope.

Perpendicular Property: When two lines are perpendicular, their slopes are opposite reciprocals of one another. The product of their slopes is –1.

LINEAR EQUATIONS

Example: Find the equation of a line passing through $(6,10)$ and $(-8,\ 3)$.

Find the **slope** of the line through the points.

$$m=\frac{y_2-y_1}{x_2-x_1}$$

$$m=\frac{3-10}{-8-6}$$

$$m=\frac{-7}{-14}$$

$$m=\frac{1}{2}$$

Use the **point-slope form** to find the equation of a line with $m=\frac{1}{2}$ and using one of the points.

Using $(6,10)$ and $m=\frac{1}{2}$, substitute into

$$y-y_1=m(x-x_1)$$

$$y-10=\frac{1}{2}(x-6)$$

$$y-10=\frac{1}{2}x-3$$

Answer: $\boldsymbol{y=\frac{1}{2}x+7}$ This is the equation of the line in **slope-intercept form**.

LINEAR EQUATIONS

Example: Find the equation of a line through $(9,2)$ and **parallel** to $2x-3y=12$.

Find the **slope** of the line.

$$2x-3y=12$$
$$y=\frac{2}{3}x-4$$
$$m=\frac{2}{3}$$

This equation is in **standard form**. Write the equation in **slope-intercept form**.

A line **parallel** to this line will have the same slope, $m=\frac{2}{3}$

Use $m=\frac{2}{3}$ and point (9, 2) to write the equation of the line.

$$y-y_1=m\left(x-x_1\right)$$

This is the **point-slope form** of a line.

$$y-2=\frac{2}{3}\left(x-9\right)$$

Answer: $\boldsymbol{y=\frac{2}{3}x-4}$

This is the **slope-intercept form** of a line.

Example: Find the equation of a line through (9, 2) and **perpendicular** to $2x-3y=12$.

From the above example, the line $2x-3y=12$ has $m=\frac{2}{3}$.

A line **perpendicular** to this line will have a slope of $m=-\frac{3}{2}$ (opposite reciprocal).

Use $m=-\frac{3}{2}$ and point (9, 2) to write the equation of the line.

$$y-2=-\frac{3}{2}\left(x-9\right)$$

Answer: $\boldsymbol{y=-\frac{3}{2}x+\frac{31}{2}}$

LINEAR EQUATIONS

Problems	Answers
1. Put the following in slope-intercept form.	
a. $8x-4y=4$	a. $y=2x-1$
b. $2x+3y=3$	b. $y=-\frac{2}{3}x+1$
2. Find the slope.	
a. $(3,4)$ and $(7,9)$	a. $\frac{5}{4}$
b. $(-2,1)$ and $(3,-3)$	b. $-\frac{4}{5}$
3. Find the equation of the line through $(5,3)$ and parallel to $y=-2x+1$	$y=-2x+13$
4. Find the equation of the line through $(-1,-3)$ and perpendicular to $y=4x-2$.	$y=-\frac{1}{4}x-\frac{13}{4}$

LINEAR EQUATIONS

Find the equation of the line that satisfies the given conditions. Write the equation in both standard form and slope-intercept form.

	Answers	
	Standard Form	Slope-Intercept Form
5. slope = $\frac{1}{2}$ line passes through $(3,4)$	$x-2y=-5$	$y=\frac{1}{2}x+\frac{5}{2}$
6. slope = $-\frac{5}{6}$ line passes through $(0,0)$	$5x+6y=0$	$y=-\frac{5}{6}x$
7. slope = 1 y-intercept –3	$x-y=3$	$y=x-3$
8. horizontal line through $(1,4)$	$y=4$	$y=0x+4$
9. slope is undefined and passing through $(-5,6)$	$x=-5$	cannot be written in the slope-intercept form.
10. slope = $-\frac{3}{2}$ x-intercept –5	$3x+2y=-15$	$y=\frac{-3}{2}x-\frac{15}{2}$
11. horizontal line through $(5,-3)$	$y=-3$	$y=0x-3$

LINEAR EQUATIONS

Find the equation of the line that satisfies the given conditions. Write the equation in both standard form and slope-intercept form.

	Answers	
	Standard Form	Slope-Intercept Form
12. vertical line passing through $(5,-4)$	$x=5$	Cannot be written in the slope-intercept form.
13. line passing through $(1,2)$ and $(5,4)$	$x-2y=-3$	$y=\frac{1}{2}x+\frac{3}{2}$
14. line passing through $(-3,4)$ and $(5,-1)$	$5x+8y=17$	$y=\frac{-5}{8}x+\frac{17}{8}$
15. line passing through $(4,-3)$ and $(4,-7)$	$x=4$	Cannot be written in the slope-intercept form.
16. line parallel to $3x+y=6$ and passing through $(1,2)$	$3x+y=5$	$y=-3x+5$
17. line passing through $(5,-3)$ and parallel to $y-2=0$	$y=-3$	$y=0x-3$
18. line perpendicular to $2x+5y=3$ and passing through $(1,7)$	$5x-2y=-9$	$y=\frac{5}{2}x+\frac{9}{2}$

FUNCTIONS

Recall that **relation** is a set of ordered pairs and that a **function** is a special type of relation.

A **function** is a set of ordered pairs (a relation) in which to each first component, there corresponds exactly one second component.

The set of first components is called the **domain of the function** and the set of second components is called **the range of the function.**

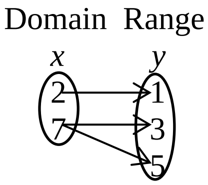
Thus, the definition of function can be restated as:

A **function** is a special type of relation. If each element of the domain corresponds to exactly one element of the range, then the relation is a function.

Examples: Determine if the following are functions.

Domain	Range
x	y
-4	0
-8	1
-16	2

This is a function, since for each first component, there is exactly one second component.



This is not a function since for the first component, 7, there are two different second components, 3 and 5.

Examples: Find the domain of the following functions.

1. $y=\dfrac{5}{x+3}$

The denominator of a rational expression $\neq 0$. Since $x+3\neq 0,\ x\neq -3.$ The domain can be written as $(-\infty,-3)\cup(-3,\infty).$

The domain is $(-\infty,-3)\cup(-3,\infty)$.

2. $y=x-2$

Since this is a linear equation, there is an infinite number of values for *x*, without restriction. The domain is any real number (*x* is any real number).

The domain is $(-\infty,\infty)$.

3. $y=\sqrt{x+2}$

The radicand of a square root must be ≥ 0. Since $x+2\geq 0,\ x\geq -2.$ The domain can be written as $[-2,\infty).$

The domain is $[-2,\infty)$.

FUNCTIONS

Examples: Find the range of the following functions.

1. $y = x^2$

Since x is squared, the corresponding value for y must be ≥ 0. The range, $y \geq 0$, can be written as $[0, \infty)$.

The range is $[0,\infty)$.

2. $y = x - 2$

Since this is a linear equation, there is an infinite number of values for *y*, without restriction. The range is any real number (*y* is any real number).

The range is $(-\infty,\infty)$.

3. $y = \sqrt{x+2}$

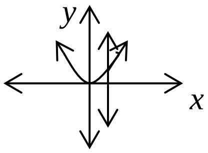
Since the square root yields an answer that is non-negative, $y \geq 0$. The range, $y \geq 0$, can be written as $[0, \infty)$.

The range is $[0,\infty)$

Vertical Line Tests: A graph in the plane represents a function if no vertical line intersects the graph at more than one point.

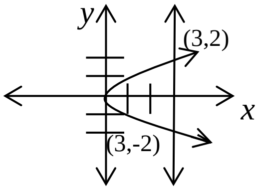
Examples: Use the Vertical Line Test to determine if the graph represents a function.

1.



This is a graph of a function. It passes the vertical line test.

2.



This is not a graph of a function. For example, the *x* value of 3 is assigned two different *y* values, 2 and -2.

Since we will often work with sets of ordered pairs of the form (*x*,*y*), it is helpful to define a function using the variables *x* and *y*.

Given a relation in *x* and *y*, if to each value of *x* in the domain there corresponds exactly one value of *y* in the range, then *y* is said to be a **function of *x*.**

FUNCTIONS

Notation: To denote that *y* is a function of *x*, we write $y = f(x)$. The expression "$f(x)$" is read *f* of *x*. It does not mean *f* times *x*. Since *y* and $f(x)$ are equal, they can be used interchangeably. This means we can write $y = x^2$, or we can write $f(x) = x^2$.

Evaluate: To evaluate or calculate a function, replace the *x* in the function rule by the given *x* value from the domain and then compute according to the rule.

Example:

1. Given: $f(x) = 6x + 5$

   Find: $f(2)$, $f(0)$, $f(-1)$

   $f(2) = 6(2) + 5 = 17$

   $f(0) = 6(0) + 5 = 5$

   $f(-1) = 6(-1) + 5 = -1$

2. Given: $g(x) = 3x^2 - 5x + 8$

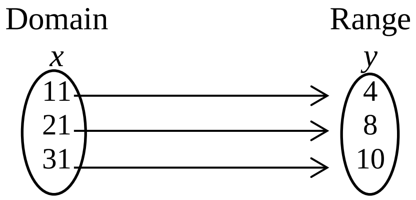
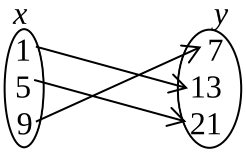
   Find: $g(1)$, $g(0)$, $g(-1)$

   $g(1) = 3(1)^2 - 5(1) + 8 = 6$

   $g(0) = 3(0)^2 - 5(0) + 8 = 8$

   $g(-1) = 3(-1)^2 - 5(-1) + 8 = 16$

FUNCTIONS

<table>
<tr><th>Problems</th><th>Answers</th></tr>
<tr><td colspan="2">1. Determine whether or not each relation defines a function. If no, explain why not.</td></tr>
<tr><td>a.

</td><td>a. Yes, for each first component, there corresponds exactly one second component</td></tr>
<tr><td>b.
<table>
<tr><th>Domain
x</th><th>Range
y</th></tr>
<tr><td>3</td><td>2</td></tr>
<tr><td>5</td><td>6</td></tr>
<tr><td>3</td><td>7</td></tr>
</table></td><td>b. No, for each first component, 3, there corresponds two different second components, 2 and 7.</td></tr>
<tr><td>c. $\{(1,3),(2,-4),(1,0)\}$</td><td>c. No, for the first component, 1, there corresponds two different second components, 3 and 0.</td></tr>
<tr><td>d. $\{(4,3),(-2,3),(1,3)\}$</td><td>d. Yes, for each first component there corresponds exactly one second component.</td></tr>
<tr><td colspan="2">2. Determine the domain and range of each of the following:</td></tr>
<tr><td>a. $\{(-3,4),(4,2),(0,0),(-2,7)\}$</td><td>a. Domain: $\{-3,-2,0,4\}$
Range: $\{0,2,4,7\}$</td></tr>
<tr><td>b.
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>–5</td><td>2</td></tr>
<tr><td>–7</td><td>4</td></tr>
<tr><td>–9</td><td>6</td></tr>
</table></td><td>b. Domain: $\{-5,-7,-9\}$
Range: $\{2,4,6\}$</td></tr>
<tr><td>c.

</td><td>c. Domain: $\{1,5,9\}$
Range: $\{7,13,21\}$</td></tr>
</table>

FUNCTIONS

Problems	Answers
3. Given $f(x)=x+2$ and $g(x)=x-3$, find the following:	
a. $f(-2)$	a. 0
b. $f(-4)$	b. –2
c. $g(0)$	c. –3
d. $g(-2)$	d. −5
4. Given $k(x)=x^2$ and $h(x)=5x-2$, find the following:	
a. $k(3)$	a. 9
b. $k(-5)$	b. 25
c. $h(3)$	c. 13
d. $h(-7)$	d. -37

ALGEBRA AND COMPOSITION OF FUNCTIONS

OPERATIONS ON FUNCTIONS

If the domains and ranges of functions *f* and *g* are subsets of the real numbers, then:

The **sum** of *f* and *g*, denoted as ***f* + *g***, is defined by

$$(f+g)(x)=f(x)+g(x)$$

The **difference** of *f* and *g*, denoted as ***f* – *g***, is defined by

$$(f-g)(x)=f(x)-g(x)$$

The **product** of *f* and *g*, denoted as ***f* • *g***, is defined by

$$(f\bullet g)(x)=f(x)g(x)$$

The **quotient** of *f* and *g*, denoted as ***f* / *g***, is defined by

$$(f/g)(x)=\frac{f(x)}{g(x)} \text{ where } g(x)\neq 0$$

The domain of each of the above functions is the set of real numbers *x* that are in the domain of both *f* and *g*. In the case of the quotient, there is the further restriction that $g(x)\neq 0$.

The **composite function** $\boldsymbol{f\circ g}$ is defined by

$$(f\circ g)(x)=f(g(x))$$

ALGEBRA AND COMPOSITION OF FUNCTIONS

Problems	Answers
Let $f(x) = 6x^2 - 5$ and $g(x) = x^2 + 2$	
Find:	
$(f+g)(x) =$	$7x^2 - 3$
$(f-g)(x) =$	$5x^2 - 7$
$(f \bullet g)(x) =$	$6x^4 + 7x^2 - 10$
$\frac{f}{g}(x) =$	$\frac{6x^2 - 5}{x^2 + 2}$
$f(g(x)) =$	$6x^4 + 24x^2 + 19$
$(f \circ g)(x) =$	$6x^4 + 24x^2 + 19$
$g(f(x)) =$	$36x^4 - 60x^2 + 27$
$(g \circ f)(x) =$	$36x^4 - 60x^2 + 27$

EXPONENTS

LAWS OF EXPONENTS

Definition: $a^n = \underbrace{a \bullet a \bullet a \bullet \ldots\, a}_{\text{n times}}$

$a =$ the base

$n =$ the exponent

Properties:

Product Rule: $a^m a^n = a^{m+n}$ **Example**: $a^2 a^3 = \underbrace{a \bullet a}_{\text{2 times}} \bullet \underbrace{a \bullet a \bullet a}_{\text{3 times}} = a^{2+3} = a^5$

Quotient Rule: $\frac{a^m}{a^n} = a^{m-n}$ **Example:** $\frac{x^8}{x^2} = x^{8-2} = x^6$

Power Rule: $\left(a^n\right)^m = a^{nm}$ **Example:** $\left(y^3\right)^4 = y^{3\bullet 4} = y^{12}$

Raising a product to a power: $(ab)^n = a^n b^n$

Raising a quotient to a power: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Zero Power Rule: $a^0 = 1$ **Example:** $(x)^0 = 1;\ (3x)^0 = 3^0 \bullet x^0 = 1 \bullet 1 = 1$

Examples: Simplify

1. $\left(12ab^2\right)\left(3a^3b^5c\right) = 36a^{1+3}b^{2+5}c = 36a^4b^7c$

2. $\left(\frac{2x^2}{3y^3}\right)^4 = \frac{2^4 x^{2\bullet 4}}{3^4 y^{3\bullet 4}} = \frac{16x^8}{81y^{12}}$

3. $3x^2y^2\left(2xy + 5y^2\right) = \left(3x^2y^2\right)\left(2xy\right) + \left(3x^2y^2\right)\left(5y^2\right)$

$= 6x^{2+1}y^{2+1} + 15x^2y^{2+2}$

$= 6x^3y^3 + 15x^2y^4$

EXPONENTS

NEGATIVE EXPONENTS

Definitions:

$$\left.\begin{array}{l} a^{-m} = \dfrac{a^{-m}}{1} \searrow \dfrac{1}{a^{m}} \\[2em] \dfrac{1}{a^{-m}} \nearrow \dfrac{a^{m}}{1} = a^{m} \\[2em] \dfrac{a^{-m}}{b^{-n}} \quad \nearrow\!\!\!\!\!\!\searrow \quad \dfrac{b^{n}}{a^{m}} \end{array}\right\}$$

Note: The act of moving a factor from the numerator to the denominator (or from the denominator to the numerator) changes a negative exponent to a positive exponent.

Examples: Simplify

1. $10^{-4} = \dfrac{10^{-4}}{1} \searrow \dfrac{1}{10^{4}} = \dfrac{1}{10,000}$

2. $ab^{-2} = \dfrac{ab^{-2}}{1} = \dfrac{a^{1}b^{-2}}{1} \searrow \dfrac{a^{1}}{b^{2}} = \dfrac{a}{b^{2}}$

Since b^{-2} has a negative exponent, move it to the denominator to become b^{2}

3. $\dfrac{x^{-5}y^{2}}{z^{-3}} = = \dfrac{y^{2}z^{3}}{x^{5}}$

Only the factors with negative exponents are moved.

4. $\left(\dfrac{-12x^{-2}y^{3}}{3x^{4}y^{2}}\right)^{-2} = \left(\dfrac{-12y^{3}}{3x^{2}x^{4}y^{2}}\right)^{-2}$

Within the parenthesis, move the factors with negative exponents.

$$= \left(\frac{-4y}{x^{6}}\right)^{-2}$$

Simplify within the parenthesis.

$$\left.\begin{array}{l} = \dfrac{(-4)^{-2}\,y^{-2}}{\left(x^{6}\right)^{-2}} \\[2em] = \dfrac{(-4)^{-2}\,y^{-2}}{x^{-12}} \end{array}\right\}$$

Use the power rule for exponents.

$$= \frac{x^{12}}{(-4)^{2}\,y^{2}}$$

Move all factors with negative exponents.

$$= \frac{x^{12}}{16y^{2}}$$

Simplify.

EXPONENTS

RATIONAL EXPONENTS

Examples: Simplify

1. $5^{1/3} \cdot 5^{1/3} = 5^{1/3+1/3} = 5^{2/3}$

2. $3^{1/2} \cdot 3^{1/3} = 3^{1/2+1/3} = 3^{3/6+2/6} = 3^{5/6}$

3. $\dfrac{6^{1/2}}{6^{1/4}} = 6^{1/2-1/4} = 6^{2/4-1/4} = 6^{1/4}$

4. $\left(x^{4/3}\right)^{1/2} = x^{\left(\frac{4}{3} \bullet \frac{1}{2}\right)} = x^{2/3}$

5. $\left(2^3 a^{15} b^{21}\right)^{1/3} = \left(2^3\right)^{1/3} \left(a^{15}\right)^{1/3} \left(b^{21}\right)^{1/3} = 2a^5 b^7$

6. $$\frac{x^{\frac{5}{3}} y^2}{x^{\frac{2}{3}} y^{\frac{12}{5}}} = x^{\frac{5}{3}-\frac{2}{3}} y^{2-\frac{12}{5}}$$

$$= x^{\frac{3}{3}} y^{-\frac{2}{5}}$$

$$= \frac{x}{y^{\frac{2}{5}}}$$

EXPONENTS

Problems	Answers
Simplify	
1. $a^{1/3} \cdot a^{2/3}$	a
2. $x^{1/2} \cdot x^{5/4}$	$x^{7/4}$
3. $5^{3/2} \cdot 5^{-1/2}$	5
4. $\left(a^{2/3}\right)^{4/5}$	$a^{8/15}$
5. $\left(x^{3/4}\right)^{4}$	x^{3}
6. $\left(a^{-3/4}\right)^{-1/3}$	$a^{1/4}$
7. $\left(16y^{4}\right)^{3/4}$	$8y^{3}$
8. $\left(a^{3}b\right)^{2/3}$	$a^{2}b^{2/3}$
9. $\dfrac{xy^{3/4}}{x^{1/2}y^{1/4}}$	$x^{1/2}y^{1/2}$

RADICALS

Definition of Square Root of *a*

The square root of the number a is b, $\sqrt{a} = b$, if $b^2 = a$.

The **radical symbol,** $\sqrt{\ }$**,** represents the principal or positive square root. The number or variable expression under the radical symbol is called the **radicand**.

Examples: Simplify

$\sqrt{25} = 5$ Check: $(5)^2 = 25$

$\sqrt{x^2} = x$ Check: $(x)^2 = x^2$

$\sqrt{x^6} = x^3$ Check: $(x^3)^2 = x^6$

$\sqrt{-4}$ is not a real number

If "a" is a negative real number, $\sqrt{a}$ is not a real number.

Properties:

1. Product Rule: $\sqrt{a}\sqrt{b} = \sqrt{ab}$ *and* $\sqrt{ab} = \sqrt{a}\sqrt{b}$

Examples: Simplify

$\sqrt{3}\sqrt{2} = \sqrt{6}$

$\sqrt{8} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$

2. Quotient Rule: $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$ and $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$

Examples: Simplify

$$\sqrt{\frac{9}{4}} = \frac{\sqrt{9}}{\sqrt{4}} = \frac{3}{2}$$

$$\frac{\sqrt{8}}{\sqrt{32}} = \sqrt{\frac{8}{32}} = \sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2}$$

RADICALS

Problems		Answers
Use the product rule and the quotient rule to simplify each radical. All variables represent positive real numbers.		
1.	$\sqrt{45}$	$3\sqrt{5}$
2.	$\sqrt{64}$	8
3.	$\sqrt{75}$	$5\sqrt{3}$
4.	$10\sqrt{27}$	$30\sqrt{3}$
5.	$\sqrt{50}\sqrt{20}$	$10\sqrt{10}$
6.	$\sqrt{12}\sqrt{48}$	24
7.	$\dfrac{\sqrt{72}}{\sqrt{8}}$	3
8.	$\dfrac{15\sqrt{10}}{5\sqrt{2}}$	$3\sqrt{5}$
9.	$\sqrt{x^4}$	x^2
10.	$\sqrt{y^3}$	$y\sqrt{y}$
11.	$\sqrt{x^4 y^8}$	$x^2 y^4$
12.	$\sqrt{\dfrac{16}{x^2}}$	$\dfrac{4}{x}$
13.	$\sqrt{\dfrac{100}{m^4}}$	$\dfrac{10}{m^2}$
14.	$\sqrt{\dfrac{75}{y^6}}$	$\dfrac{5\sqrt{3}}{y^3}$

RADICALS

Definition: ***n*^*th* root of *a***.

The n^{th} root of the number a is b, $\sqrt[n]{a}=b$ if $b^n=a$.

For $\sqrt[n]{a}$, n is the **index number** and a is the **radicand**.

Examples: Simplify

$\sqrt[3]{-8}=-2$ Check: $(-2)^3=-8$

$\sqrt[4]{16}=2$ Check: $(2)^4=16$

The **product and quotient rules** for radicals apply for all index numbers 2 and larger (for all $n\geq 2$).

$$\sqrt[n]{a}\cdot\sqrt[n]{b}=\sqrt[n]{ab} \text{ and } \sqrt[n]{ab}=\sqrt[n]{a}\cdot\sqrt[n]{b}$$

$$\sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}} \qquad \text{and} \quad \frac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\frac{a}{b}}$$

Examples: Simplify

1. $\sqrt[3]{128}=\sqrt[3]{64}\cdot\sqrt[3]{2}$ Notice that 64 is a perfect cube. That is, $64=4^3$.

$$=4\sqrt[3]{2}$$

2. $\sqrt[4]{80}=\sqrt[4]{16}\cdot\sqrt[4]{5}$ Notice that 16 is a perfect 4th. That is, $16=2^4$.

$$=2\sqrt[4]{5}$$

3. $\sqrt[3]{(x+y)^5}=\sqrt[3]{(x+y)^3}\cdot\sqrt[3]{(x+y)^2}$

$$=(x+y)\cdot\sqrt[3]{(x+y)^2}$$

4. $\sqrt[5]{x^{11}y^7z^{20}}=\sqrt[5]{x^{10}xy^5y^2z^{20}}$

$$=\sqrt[5]{x^{10}y^5z^{20}}\cdot\sqrt[5]{xy^2}$$

$$=x^2yz^4\sqrt[5]{xy^2}$$

Notice that x^{10}, y^5, and z^{20} are all perfect 5ths. That is, $x^{10}=\left(x^2\right)^5$, $y^5=(y)^5$, and $z^{20}=\left(z^4\right)^5$.

RADICALS

Problems	Answers
Simplify. All variables represent positive real numbers.	
1. $\sqrt[3]{216x^6}$	$6x^2$
2. $\sqrt[3]{432}$	$6\sqrt[3]{2}$
3. $\sqrt[4]{81y^5}$	$3y\sqrt[4]{y}$
4. $\sqrt[7]{\left(3x+y\right)^{10}}$	$\left(3x+y\right)\sqrt[7]{\left(3x+y\right)^3}$
5. $\sqrt[5]{32a^{11}b^{15}c^{12}}$	$2a^2b^3c^2\sqrt[5]{ac^2}$
6. $\sqrt[3]{\dfrac{64}{b^6}}$	$\dfrac{4}{b^2}$
7. $\sqrt[3]{\dfrac{y^6}{27x^3}}$	$\dfrac{y^2}{3x}$

RADICALS

ADDITION/SUBTRACTION OF RADICALS

To have **like radicals**, the following must be true:

1. The radicals must have the same index.
2. The radicands must be the same.

To combine like radicals (i.e. add or subtract);

1. Perform any simplification within the terms.
2. Use the distributive property to combine terms that have like radicals.

Examples: Perform the operations. Simplify.

1.
$$\begin{aligned} 6\sqrt{3}+2\sqrt{3} &= (6+2)\sqrt{3} \\ &= 8\sqrt{3} \end{aligned}$$

2.
$$\begin{aligned} 14\sqrt[5]{2}-6\sqrt[5]{2} &= (14-6)\sqrt[5]{2} \\ &= 8\sqrt[5]{2} \end{aligned}$$

3.
$$\begin{aligned} 2\sqrt{6}+8\sqrt{6}-3\sqrt{6} &= (2+8-3)\sqrt{6} \\ &= 7\sqrt{6} \end{aligned}$$

These radicals are **like radicals**.
Add or subtract the coefficients,
using the distributive property.

4.
$$\begin{aligned} 8\sqrt{27}-\sqrt{3} &= 8\sqrt{9\cdot 3}-\sqrt{3} \\ &= 8\cdot 3\sqrt{3}-\sqrt{3} \\ &= 24\sqrt{3}-1\sqrt{3} \\ &= 23\sqrt{3} \end{aligned}$$

5.
$$\begin{aligned} 9\sqrt{12}+16\sqrt{27} &= 9\sqrt{4\cdot 3}+16\sqrt{9\cdot 3} \\ &= 9\cdot 2\sqrt{3}+16\cdot 3\sqrt{3} \\ &= 18\sqrt{3}+48\sqrt{3} \\ &= 66\sqrt{3} \end{aligned}$$

6.
$$\begin{aligned} 4\sqrt{3x^3}-\sqrt{12x} &= 4\sqrt{3\cdot x^2\cdot x}-\sqrt{4\cdot 3\cdot x} \\ &= 4x\sqrt{3x}-2\sqrt{3x} \\ &= (4x-2)\sqrt{3x} \end{aligned}$$

7.
$$\begin{aligned} \sqrt[3]{54x}-\sqrt[3]{2x^4} &= \sqrt[3]{27\cdot 2x}-\sqrt[3]{2x^3\cdot x} \\ &= 3\sqrt[3]{2x}-x\sqrt[3]{2x} \\ &= (3-x)\sqrt[3]{2x} \end{aligned}$$

Create **like radicals** first.
Then combine **like radicals**.

RADICALS

Problems	Answers
Perform the operations. Simplify.	
1. $\sqrt{5}-3\sqrt{5}+6\sqrt{5}$	$4\sqrt{5}$
2. $5\sqrt{2}-6\sqrt{2}+10\sqrt{2}$	$9\sqrt{2}$
3. $2\sqrt{45}-5\sqrt{20}$	$-4\sqrt{5}$
4. $5\sqrt{18}-3\sqrt{32}$	$3\sqrt{2}$
5. $8\sqrt{5}-\sqrt{20}$	$6\sqrt{5}$
6. $\sqrt{8x}+\sqrt{32x}$	$6\sqrt{2x}$
7. $\sqrt{28x^3}+\sqrt{63x^3}$	$5x\sqrt{7x}$
8. $\sqrt{72xy}-\sqrt{200xy}$	$-4\sqrt{2xy}$
9. $\sqrt{54x^2y}-\sqrt{24x^2y}$	$x\sqrt{6y}$
10. $\sqrt{100x^2y^2}-\sqrt{81x^2y^2}$	xy

RADICALS

	Problems	Answers
11.	$\sqrt{40a^5}$	$2a^2\sqrt{10a}$
12.	$\sqrt{16x^3y^8}$	$4xy^4\sqrt{x}$
13.	$\sqrt{24r^9s^{20}t^{17}}$	$2r^4s^{10}t^8\sqrt{6rt}$
14.	$\sqrt[3]{48a^{11}}$	$2a^3\sqrt[3]{6a^2}$
15.	$\sqrt[3]{54xy^8z^{15}}$	$3y^2z^5\sqrt[3]{2xy^2}$
16.	$\sqrt[4]{16b^{18}}$	$2b^4\sqrt[4]{b^2}$
17.	$\sqrt[4]{162a^7b^{12}c^{21}}$	$3ab^3c^5\sqrt[4]{2a^3c}$
18.	$\sqrt{18}+\sqrt{50}$	$8\sqrt{2}$
19.	$5\sqrt{24}+2\sqrt{54}$	$16\sqrt{6}$
20.	$3\sqrt{2x}-\sqrt{8x}+5\sqrt{18x}$	$16\sqrt{2x}$
21.	$\left(\sqrt{x}+5\right)\left(\sqrt{x}-5\right)$	$x-25$
22.	$\left(3-2\sqrt{a}\right)\left(3+2\sqrt{a}\right)$	$9-4a$
23.	$\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)$	3

RATIONAL EXPONENTS

Definition of a Rational Exponent

$$a^{\frac{m}{n}} = \begin{matrix} \nearrow \sqrt[n]{a^m} \\ \searrow \left(\sqrt[n]{a}\right)^m \end{matrix}$$

Examples: Simplify

$$16^{\frac{1}{2}} = \sqrt{16} = 4$$

$$(-8)^{\frac{1}{3}} = \sqrt[3]{-8} = -2$$

$$-64^{\frac{3}{2}} = -\left(\sqrt{64}\right)^3 = -(8)^3 = -512$$

$$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = (3)^2 = 9$$

$$125^{-\frac{1}{3}} = \frac{1}{125^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{125}} = \frac{1}{5}$$

$$\left.\begin{aligned} &\left(a^6b^9c^3\right)^{\frac{4}{3}} = \left(a^6\right)^{\frac{4}{3}}\left(b^9\right)^{\frac{4}{3}}\left(c^3\right)^{\frac{4}{3}} = a^8b^{12}c^4 \\ &\left(8x^3y^6\right)^{-\frac{2}{3}} = \frac{1}{\left(8x^3y^6\right)^{\frac{2}{3}}} = \frac{1}{8^{\frac{2}{3}}\left(x^3\right)^{\frac{2}{3}}\left(y^6\right)^{\frac{2}{3}}} = \frac{1}{4x^2y^4} \end{aligned}\right\} \text{Use Power Rule of Exponents: } (ab)^n = a^nb^n$$

RATIONAL EXPONENTS

Problems	Answers
Simplify	
1. $8^{\frac{2}{3}}$	4
2. $27^{\frac{4}{3}}$	81
3. $-16^{\frac{3}{2}}$	-64
4. $25^{-\frac{3}{2}}$	$\frac{1}{125}$
5. $4^{\frac{5}{2}}$	32
6. $36^{-\frac{3}{2}}$	$\frac{1}{216}$
7. $81^{\frac{3}{4}}$	27
8. $\left(a^6b^8\right)^{\frac{1}{2}}$	a^3b^4
9. $\left(x^{24}b^{20}\right)^{\frac{3}{4}}$	$x^{18}b^{15}$
10. $\left(8a^{18}b^{33}\right)^{-\frac{2}{3}}$	$\frac{1}{4a^{12}b^{22}}$
11. $\left(16p^{-16}q^{28}\right)^{-\frac{1}{4}}$	$\frac{p^4}{2q^7}$

RADICAL EQUATIONS

To solve a Radical Equation:

1. Isolate one radical expression on one side of the equation.
2. Raise both sides of the equation to the power that is the same as the index of the radical.
3. Solve the resulting equation. If it still contains a radical, go back to step 1.
4. Check the results to eliminate extraneous solutions.

Example: Solve $\sqrt{2x-3}-x=-9$

$$\sqrt{2x-3}=x-9$$

Isolate the radical.

$$\left(\sqrt{2x-3}\right)^2=\left(x-9\right)^2$$

The index number is 2, so square both sides.

$$\left.\begin{array}{l} 2x-3=x^2-18x+81 \\ 0=x^2-20x+84 \\ 0=\left(x-14\right)\left(x-6\right) \\ x=14 \qquad x=6 \end{array}\right\} \text{Solve.}$$

Check $x=14$

$$\begin{array}{r|l} \multicolumn{2}{c}{\sqrt{2x-3}-x=-9} \\ \hline \sqrt{2\left(14\right)-3}-14 & -9 \\ \sqrt{28-3}-14 & \\ \sqrt{25}-14 & \\ 5-14 & \\ \multicolumn{2}{c}{-9=-9} \end{array}$$

Check $x=6$

$$\begin{array}{r|l} \multicolumn{2}{c}{\sqrt{2x-3}-x=-9} \\ \hline \sqrt{2\left(6\right)-3}-6 & -9 \\ \sqrt{12-3}-6 & \\ \sqrt{9}-6 & \\ 3-6 & \\ \multicolumn{2}{c}{-3\neq-9} \end{array}$$

Check for extraneous solutions.

$x=14$ is a solution and
$x=6$ is an extraneous solution.

Answer: $\mathbf{x=14}$

RADICAL EQUATIONS

Example: Solve $\sqrt{x+7}-\sqrt{x}=1$

$$\sqrt{x+7}=1+\sqrt{x}$$

Isolate one radical

$$\left(\sqrt{x+7}\right)^2=\left(1+\sqrt{x}\right)^2$$

The index number is 2, so square both sides.

$$x+7=1+2\sqrt{x}+x$$

$$\left.\begin{aligned} 6&=2\sqrt{x} \\ 3&=\sqrt{x} \end{aligned}\right\}$$

Isolate the radical.

$$3^2=\left(\sqrt{x}\right)^2$$

The index number is 2, so square both sides.

$$9=x$$

Check $x=9$

$$\begin{array}{rc|l}
\sqrt{x+7}-\sqrt{x} & = & 1 \\
\hline
\sqrt{9+7}-\sqrt{9} & & 1 \\
\sqrt{16}-\sqrt{9} & & \\
4-3 & & \\
1 & = & 1
\end{array}$$

Answer: $\mathbf{x=9}$

RADICAL EQUATIONS

Problems	Answers
Solve.	
1. $\sqrt{5x-4}=x$	$x=4,\ x=1$
2. $\sqrt[3]{x^3+x^2-1}=x$	$x=-1,\ x=1$
3. $\sqrt{9-x}-3=x$	$x=0$ (–7 is extraneous)
4. $\sqrt{2x+1}+7=5$	No solution $\left(\frac{3}{2} \text{ is extraneous}\right)$
5. $2=\sqrt[3]{3x-4}$	$x=4$
6. $\sqrt[4]{2x+7}=3$	$x=37$

COMPLEX NUMBERS

The **imaginary number *i*** is defined as $i = \sqrt{-1}$.

From the definition, it follows that $i^2 = -1$.

Example: Simplify $\sqrt{-9}$

$$\begin{aligned} \sqrt{-9} &= \sqrt{-1 \cdot 9} \\ &= \sqrt{-1} \cdot \sqrt{9} \\ &= i \cdot 3 \\ &= 3i \end{aligned}$$

Example: Simplify $\sqrt{-24}$

$$\begin{aligned} \sqrt{-24} &= \sqrt{-1 \cdot 4 \cdot 6} \\ &= \sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{6} \\ &= i \cdot 2 \cdot \sqrt{6} \\ &= 2i\sqrt{6} \end{aligned}$$

A **complex number** is any number that can be written in the form $a + bi$, where *a* and *b* are real numbers and $i = \sqrt{-1}$.

Example: Write this complex number in $a + bi$ form: $5 + \sqrt{-81}$

$$\begin{aligned} 5 + \sqrt{-81} &= 5 + \sqrt{-1 \cdot 81} \\ &= 5 + 9i \end{aligned}$$

Example: Simplify and write in $a + bi$ form: $\sqrt{-49}$

$$\begin{aligned} \sqrt{-49} &= \sqrt{-1} \cdot \sqrt{49} \\ &= 7i \\ &= 0 + 7i \end{aligned}$$

COMPLEX NUMBERS

Write each imaginary number in terms of *i*.

Problems	Answers
1. $\sqrt{-25}$	$5i$
2. $\sqrt{-32}$	$4i\sqrt{2}$ or $4\sqrt{2}\ i$
3. $-\sqrt{-108}$	$-6i\sqrt{3}$ or $-6\sqrt{3}\ i$
4. $\sqrt{-\frac{36}{81}}$	$\frac{2}{3}i$

Write each complex number in the form $a+bi$.

5. $3-\sqrt{-16}$	$3-4i$
6. $5+\sqrt{-27}$	$5+3i\sqrt{3}$ or $5+3\sqrt{3}\ i$
7. $\frac{9+13i}{7}$	$\frac{9}{7}+\frac{13}{7}i$
8. 15	$15+0i$
9. $\sqrt{-30}$	$0+i\sqrt{30}$ or $0+\sqrt{30}\ i$
10. $6-\sqrt{-54}$	$6-3i\sqrt{6}$ or $6-3\sqrt{6}\ i$

COMPLEX NUMBERS

OPERATIONS WITH COMPLEX NUMBERS

Example: Perform this addition. Write the answer in $a+bi$ form.
$\left(3+\sqrt{-25}\right)+\left(5-\sqrt{-4}\right)$

$$\begin{aligned}\left(3+\sqrt{-25}\right)+\left(5-\sqrt{-4}\right)&=(3+5i)+(5-2i)\\&=(3+5)+(5i-2i)\\&=8+3i\end{aligned}$$

$\sqrt{-1}=i$

Combine "like" term.

Example: Perform this multiplication. Write the answer in $a+bi$ form.
$\left(3-\sqrt{-16}\right)\left(4+\sqrt{-4}\right)$

$$\begin{aligned}\left(3-\sqrt{-16}\right)\left(4+\sqrt{-4}\right)&=(3-4i)(4+2i)\\&=12+6i-16i-8i^2\\&=12-10i-8i^2\\&=12-10i-8(-1)\\&=20-10i\end{aligned}$$

$\sqrt{-1}=i$

Distribute Property

Combine "like" terms.

Recall $i^2=-1$

Example: Perform this division. Write the answer in $a+bi$ form.
$\frac{5}{7i}$

$$\begin{aligned}\frac{5}{7i}&=\frac{5}{7i}\cdot\frac{i}{i}\\&=\frac{5i}{7i^2}\\&=\frac{5i}{7(-1)}\\&=0-\frac{5}{7}i\end{aligned}$$

Recall $i^2=-1$

COMPLEX NUMBERS

COMPLEX CONJUGATES

The complex numbers $a+bi$ and $a-bi$ are called **complex conjugates**.

Example: Perform this division. Write the answer in $a+bi$ form.

$$\frac{6+4i}{3-5i}$$

$$\frac{6+4i}{3-5i}=\frac{6+4i}{3-5i}\cdot\frac{3+5i}{3+5i}$$

Multiply the numerator and the denominator by the conjugate of the denominator.

$$=\frac{(6+4i)(3+5i)}{(3-5i)(3+5i)}$$

$$=\frac{18+30i+12i+20i^2}{9+15i-15i-25i^2}$$

Distributive Property

$$=\frac{18+42i+20i^2}{9-25i^2}$$

Combine “like” terms.

$$=\frac{18+42i+20(-1)}{9-25(-1)}$$

Recall $i^2=-1$

$$=\frac{-2+42i}{34}$$

$$=\frac{-2}{34}+\frac{42}{34}i$$

$a+bi$ form

$$=-\frac{1}{17}+\frac{21}{17}i$$

COMPLEX NUMBERS

Perform the indicated operations. Write the answer in $a+bi$ form.

Problems	Answers
1. $(4+5i)-(-10+3i)$	$14+2i$
2. $\left(7-\sqrt{-36}\right)+\left(9+\sqrt{-49}\right)$	$16+i$
3. $5i(8+6i)$	$-30+40i$
4. $(3+7i)(5-3i)$	$36+26i$
5. $\left(7-\sqrt{-4}\right)\left(2+\sqrt{-16}\right)$	$22+24i$
6. $\frac{6}{5i}$	$0-\frac{6}{5}i$
7. $\frac{4}{3+i}$	$\frac{6}{5}-\frac{2}{5}i$
8. $\frac{5+5i}{4+3i}$	$\frac{7}{5}+\frac{1}{5}i$
9. $\frac{3-\sqrt{-4}}{4-\sqrt{-1}}$	$\frac{14}{17}-\frac{5}{17}i$
10. $\frac{5-5i}{1+i}$	$0-5i$

SOLVING QUADRATIC EQUATIONS

SOLVING QUADRATIC EQUATIONS USING THE SQUARE ROOT PROPERTY

Example: Solve $x^2 - 18 = 0$

$$x^2 - 18 = 0$$

$$x^2 = 18$$

Isolate the perfect square.

$$\sqrt{x^2} = \pm\sqrt{18}$$

Find the square root of both sides using the Square Root Property.

$$x = \pm\sqrt{9 \cdot 2} = \pm\sqrt{3^2 \cdot 2}$$

Remember to use ± sign when finding the square roots.

$$x = \pm 3\sqrt{2}$$

The solutions are $x = 3\sqrt{2}$ and $x = -3\sqrt{2}$.

Example: Solve $(x-3)^2 = 16$

$$(x-3)^2 = 16$$

The perfect square is already isolated.

$$\sqrt{(x-3)^2} = \pm\sqrt{16}$$

Find the square root of both sides using the Square Root Property.

$$\begin{array}{ccc} & x-3 = \pm 4 & \\ \swarrow & & \searrow \\ x-3 = 4 & & x-3 = -4 \\ x = 7 & & x = -1 \end{array}$$

$x - 3 = \pm 4$ needs to be solved with two separate equations.

The solutions are $x = 7$ and $x = -1$.

SOLVING QUADRATIC EQUATIONS

Example: Solve $x^2 + 64 = 0$

$$x^2 + 64 = 0$$

$$x^2 = -64$$

Isolate the perfect square.

$$\sqrt{x^2} = \pm\sqrt{-64}$$

Remember to use the ± sign when finding the square roots.

$$x = \pm 8i$$

Recall that $\sqrt{-1} = i$.

The solutions are $x = 8i$ and $x = -8i$.

Example: Solve $16x^2 + 49 = 0$

$$16x^2 + 49 = 0$$
$$16x^2 = -49$$
$$x^2 = -\frac{49}{16}$$
$$\sqrt{x^2} = \pm\sqrt{-\frac{49}{16}}$$
$$x = \pm\frac{\sqrt{-49}}{\sqrt{16}}$$
$$x = \pm\frac{\sqrt{7^2 \cdot (-1)}}{4}$$
$$x = \pm\frac{7i}{4}$$

The solutions are $x = \frac{7}{4}i$ and $x = -\frac{7}{4}i$.

SOLVING QUADRATIC EQUATIONS

SOLVING QUADRATIC EQUATIONS USING SQUARE ROOT PROPERTY

Solve the equations. Be sure to get two solutions.

Problems	Answers
1. $x^2 = 16$	$x = \pm 4$
2. $x^2 = 25$	$x = \pm 5$
3. $x^2 = 10$	$x = \pm\sqrt{10}$
4. $x^2 = 12$	$x = \pm 2\sqrt{3}$
5. $x^2 = 9$	$x = \pm 3$
6. $x^2 = -100$	$x = \pm 10i$
7. $x^2 = -24$	$x = \pm 2i\sqrt{6}$
8. $x^2 = -50$	$x = \pm 5i\sqrt{2}$

SQUARE QUADRATIC EQUATIONS

SOLVING QUADRATIC EQUATIONS USING SQUARE ROOT PROPERTY

Problems	Answers
9. $(x-1)^2 = 36$	$x = 7$ and $x = -5$
10. $(x+2)^2 = 49$	$x = 5$ and $x = -9$
11. $(x-5)^2 = 25$	$x = 0$ and $x = 10$
12. $(x+4)^2 = 18$	$x = -4 \pm 3\sqrt{2}$
13. $(x+3)^2 = -1$	$x = -3 \pm i$
14. $(x-2)^2 = -9$	$x = 2 \pm 3i$
15. $(x-8)^2 = 60$	$x = 8 \pm 2\sqrt{15}$
16. $(x+2)^2 = -40$	$x = -2 \pm 2i\sqrt{10}$
17. $(x-3)^2 = -8$	$x = 3 \pm 2i\sqrt{2}$

SOLVING QUADRATIC EQUATIONS

SOLVING QUADRATIC EQUATIONS USING FACTORING

The Zero-Factor Property

When the product of two real numbers is 0, at least one of them is 0. If *a* and *b* represent real numbers, and

if $ab = 0$ then $a = 0$ or $b = 0$

Example: Solve using factoring $4x^2 - 7x - 2 = 0$

$$4x^2 - 7x - 2 = 0$$

Set equation equal to 0.

$$(4x+1)(x-2) = 0$$

Factor completely.

$$4x+1=0 \quad \text{or} \quad x-2=0$$

Use the Zero-Factor property to set each factor equal to 0.

$$\begin{aligned} 4x+1&=0 \\ 4x&=-1 \\ x&=-\frac{1}{4} \end{aligned} \qquad \begin{aligned} x-2&=0 \\ x&=2 \end{aligned}$$

Solve the resulting linear equations.

Check: $x = 2$

$$\begin{aligned} 4x^2 - 7x - 2 &= 0 \\ 4(2)^2 - 7(2) - 2 &= 0 \\ 16 - 14 - 2 &= 0 \\ 0 &= 0 \end{aligned}$$

Check: $x = -\frac{1}{4}$

$$\begin{aligned} 4x^2 - 7x - 2 &= 0 \\ 4\left(-\frac{1}{4}\right)^2 - 7\left(-\frac{1}{4}\right) - 2 &= 0 \\ \frac{4}{16} + \frac{7}{4} - 2 &= 0 \\ \frac{1}{4} + \frac{7}{4} - 2 &= 0 \\ 0 &= 0 \end{aligned}$$

Check the results in the original equation.

The solutions are $x = -\frac{1}{4}$ and $x = 2$.

SOLVING QUADRATIC EQUATIONS

SOLVING QUADRATIC EQUATIONS WITH THE QUADRATIC FORMULA

Example: Solve $4x^2-7x-2=0$

$$4x^2-7x-2=0$$

Set the equation equal to zero.

$$a=4,\ b=-7,\ \text{and}\ \ c=-2$$

Compare this equation with the

Standard Form $ax^2+bx+c=0$
and identify the *a*, *b* and *c* values
in this equation.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4(4)(-2)}}{2(4)}$$

Substitute the values for *a*, *b* and *c* into the Quadratic Formula.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{+7\pm\sqrt{49-4(-8)}}{8}$$

$$x=\frac{7\pm\sqrt{49+32}}{8}=\frac{7\pm\sqrt{81}}{8}$$

$$x=\frac{7\pm9}{8}$$

$$x=\frac{7+9}{8}=\frac{16}{8} \qquad\qquad x=\frac{7-9}{8}=\frac{-2}{8}$$

$$x=2 \qquad\qquad x=-\frac{1}{4}$$

Check: $x=2$

$$4x^2-7x-2=0$$
$$4(2)^2-7(2)-2=0$$
$$16-14-2=0$$
$$0=0$$

Check: $x=-\frac{1}{4}$

$$4x^2-7x-2=0$$
$$4\left(-\frac{1}{4}\right)^2-7\left(-\frac{1}{4}\right)-2=0$$
$$+\frac{4}{16}+\frac{7}{4}-2=0$$
$$+\frac{1}{4}+\frac{7}{4}-2=0$$
$$0=0$$

Check the results in the original equation.

The solutions are $x=2$ and $x=-\frac{1}{4}$.

SOLVING QUADRATIC EQUATIONS

The Discriminant: For a quadratic equation of the form $ax^2 + bx + c = 0$ with real-number coefficients and $a \neq 0,$ the expression $b^2 - 4ac$ is called the **discriminant** and can be used to determine the number and type of the solutions of the equation.

Discriminant: $b^2 - 4ac$	***Number and type of solutions***
Positive……………………	Two different real numbers
0…………………………...	One repeated solution, a rational number
Negative…………………..	Two different imaginary numbers that are complex conjugates

SOLVING QUADRATIC EQUATIONS

Problems	Answers
Solve.	
1. $x^2+3x-28=0$	$x=-7$ and $x=4$
2. $x^2-x-12=0$	$x=4$ and $x=-3$
3. $3x^2-3x-18=0$	$x=3$ and $x=-2$
4. $x^2-36=0$	$x=\pm 6$
5. $x^2=5x$	$x=0$ and $x=5$
6. $6x^2-5x=6$	$x=-\frac{2}{3}$ and $x=\frac{3}{2}$
7. $x^2-30=x$	$x=6$ and $x=-5$
8. $x^2-8x-5=0$	$x=4\pm\sqrt{21}$

SOLVING QUADRATIC EQUATIONS

Problems	Answers
Solve.	
9. $x^2+4x=1$	$x=-2\pm\sqrt{5}$
10. $x^2+x+1=0$	$x=-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$
11. $3x^2-2x=-4$	$x=\frac{1}{3}\pm\frac{\sqrt{11}}{3}i$
12. $2x^2=5-x$	$x=\frac{-1\pm\sqrt{41}}{4}=-\frac{1}{4}\pm\frac{\sqrt{41}}{4}$
13. $x^2+10x+4=0$	$x=-5\pm\sqrt{21}$
14. $1=5x^2+7x$	$x=\frac{-7\pm\sqrt{69}}{10}=-\frac{7}{10}\pm\frac{\sqrt{69}}{10}$
15. $x^2-11x-1=0$	$x=\frac{11\pm5\sqrt{5}}{2}=\frac{11}{2}\pm\frac{5\sqrt{5}}{2}$

SOLVING QUADRATIC AND RATIONAL INEQUALITIES

TO SOLVE A QUADRATIC OR RATIONAL INEQUALITIES:

1. Write the inequality in standard form with 0 on the right side of the inequality.

2. Solve the related equation. The solutions are called critical numbers.

3. Consider any restrictions on the domain. For example, in rational inequalities since the denominator cannot equal 0, set the denominator equal to 0 to determine the restrictions.

4. Locate the numbers from Steps 2 and 3 on a number line to create intervals. Determine if these numbers are included in the solution.

5. Test a number from each interval back in the original inequality. If it makes the inequality true, the interval is included in the solution.

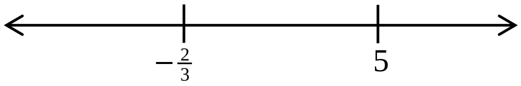
Example: Solve the inequality $3x^2 - 13x < 10$

$$3x^2 - 13x < 10$$
$$3x^2 - 13x - 10 < 0$$

Write the inequality in standard form.

$$\left.\begin{array}{l} 3x^2 - 13x - 10 = 0 \\ (3x+2)(x-5) = 0 \\ x = -\dfrac{2}{3} \qquad x = 5 \end{array}\right\}$$

Solve the related equation. (There are no restrictions on the domain).



Locate the numbers on a number line.

$$\left(-\infty, -\frac{2}{3}\right) \qquad \left(-\frac{2}{3}, 5\right) \qquad (5, \infty)$$

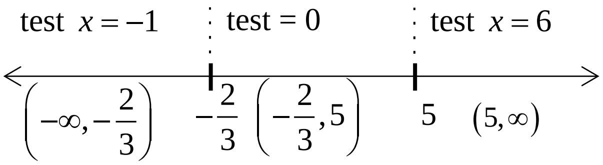
Create the intervals.

$-\dfrac{2}{3}$ and 5 are not in the solution set.

Example continues on the next page.

SOLVING QUADRATIC AND RATIONAL INEQUALITIES

Test a number from each interval into $3x^2 - 13x < 10$.



Test a number from each interval in the original inequality.

For interval $\left(-\infty, -\frac{2}{3}\right)$, test $x = -1$.

$$3x^2 - 13x < 10$$
$$3(-1)^2 - 13(-1) < 10$$
$$3 + 13 < 10$$
$$16 < 10$$

False, this interval is not included in the solution set.

For interval $\left(-\frac{2}{3}, 5\right)$, test $x = 0$.

$$3x^2 - 13x < 10$$
$$3(0)^2 - 13(0) < 10$$
$$0 < 10$$

True, this interval is included in the solution set.

For interval $(5, \infty)$, test $x = 6$.

$$3x^2 - 13x < 10$$
$$3(6)^2 - 13(6) < 10$$
$$30 < 10$$

False, this interval is not included in the solution set.

The solution is $\left(-\frac{2}{3}, 5\right)$.

SOLVING QUADRATIC AND RATIONAL INEQUALITIES

Example: Solve the inequality $\frac{x+3}{x-5} \geq 0$

$$\frac{x+3}{x-5} \geq 0$$

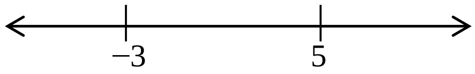
The inequality is in standard form.

$$\left.\begin{aligned} \frac{x+3}{x-5} &= 0 \\ (x-5)\bullet\frac{x+3}{x-5} &= 0\bullet(x-5) \\ x+3 &= 0 \\ x &= -3 \end{aligned}\right\}$$

Solve the related equation.

$$\begin{aligned} x-5 &\neq 0 \\ x &\neq 5 \end{aligned}$$

Consider restrictions on the domain.



Locate the numbers on a number line.

$(-\infty, -3]$ $\quad [-3, 5)$ $\quad (5, \infty)$

Create the intervals. −3 is in the solution set but 5 is not.

For interval $(-\infty, -3]$, test $x = -5$

$$\begin{aligned} \frac{x+3}{x-5} &\geq 0 \\ \frac{-5+3}{-5-5} &\geq 0 \\ \frac{1}{5} &\geq 0 \end{aligned}$$

Test a number from each interval in the original inequality.

True, this interval is included in the solution set.

Example continues on the next page.

SOLVING QUADRATIC AND RATIONAL INEQUALITIES

For interval $[-3,\ 5)$, test $x = 0$

$$\frac{x+3}{x-5} \geq 0$$

$$\frac{0+3}{0-5} \geq 0$$

$$-\frac{3}{5} \geq 0$$

False, this interval is not included in the solution set.

For interval $(5,\ \infty)$, test $x = 6$.

$$\frac{x+3}{x-5} \geq 0$$

$$\frac{6+3}{6-5} \geq 0$$

$$9 \geq 0$$

True, this interval is included in the solution set.

The solution is $(-\infty,\ -3] \cup (5,\infty)$.

SOLVING QUADRATIC AND RATIONAL INEQUALITIES

Solve the inequalities.

Problem	Answer
1. $2x^2 - 6 \le -11x$	$\left[-6, \frac{1}{2}\right]$
2. $x^2 - 2x > 8$	$(-\infty, -2) \cup (4, \infty)$
3. $\frac{x-5}{x+1} < 0$	$(-1, 5)$
4. $\frac{x^2 + x - 2}{x-3} \ge 0$	$[-2, 1] \cup (3, \infty)$

EQUATIONS THAT CAN BE WRITTEN IN QUADRATIC FORM

SOLVING EQUATIONS USING U-SUBSTITUTION

EXAMPLE: Solve for "x" $(x+3)^2-5(x+3)+6=0$

$(x+3)^2-5(x+3)+6=0$

This equation is quadratic in form.

$u^2-5u+6=0$

Substitute "u" in the equation for $x+3$ (Let $x+3=u$).

$$(u-3)(u-2)=0$$
$$u-3=0 \qquad u-2=0$$
$$u=3 \qquad u=2$$

Factor and solve.

$$x+3=3 \qquad x+3=2$$
$$x=0 \qquad x=-1$$

To solve the original equation for "x", substitute "$x+3$" in for "u" (Let $u=x+3$).

Check $x=0$

$$(x+3)^2-5(x+3)+6=0$$
$$(0+3)^2-5(0+3)+6$$
$$3^2-5(3)+6$$
$$9-15+6$$
$$-6+6$$
$$0=0$$

Check $x=-1$

$$(x+3)^2-5(x+3)+6=0$$
$$(-1+3)^2-5(-1+3)+6$$
$$2^2-5(2)+6$$
$$4-10+6$$
$$-6+6$$
$$0=0$$

The solutions are $x=0$ and $x=-1$.

EQUATIONS THAT CAN BE WRITTEN IN QUADRATIC FORM

SOLVING EQUATIONS USING U-SUBSTITUTION

Example: Solve for "x" $x^6 + 7x^3 - 8 = 0$

$$\left.\begin{aligned} &x^6 + 7x^3 - 8 = 0 \\ &\left(x^3\right)^2 + 7x^3 - 8 = 0 \end{aligned}\right\}$$

This equation is quadratic in form.

$$u^2 + 7u - 8 = 0$$

Substitute "u" in the equation for x^3 $\left(\text{Let } x^3 = u\right)$

$$\begin{aligned} &(u-1)(u+8) = 0 \\ &u - 1 = 0 \quad u + 8 = 0 \\ &\quad u = 1 \qquad u = -8 \end{aligned}$$

Factor and solve

$$\begin{aligned} &x^3 = 1 & &x^3 = -8 \\ &\sqrt[3]{x^3} = \sqrt[3]{1} & &\sqrt[3]{x^3} = \sqrt[3]{-8} \\ &\quad x = 1 & &\quad x = -2 \end{aligned}$$

To solve the original equation for "x", substitute x^3 in for "u". $\left(\text{Let } u = x^3\right)$

Check $x = 1$

$$\begin{array}{r|l} \multicolumn{2}{c}{x^6 + 7x^3 - 8 = 0} \\ \hline (1)^6 + 7(1)^3 - 8 & 0 \\ 1 + 7 - 8 & \\ 8 - 8 & \\ 0 & = 0 \end{array}$$

Check $x = -2$

$$\begin{array}{r|l} \multicolumn{2}{c}{x^6 + 7x^3 - 8 = 0} \\ \hline (-2)^6 + 7(-2)^3 - 8 & 0 \\ 64 + 7(-8) - 8 & \\ 64 - 56 - 8 & \\ 0 & = 0 \end{array}$$

The solutions are $x = 1$ and $x = -2$.

EQUATIONS THAT CAN BE WRITTEN IN QUADRATIC FORM

Problems	Answers
Solve using U-Substitution.	
1. $\left(x+6\right)^2+9\left(x+6\right)+20=0$	$x=-11$ and $x=-10$
2. $5\left(x-2\right)^2+18\left(x-2\right)-8=0$	$x=\frac{12}{5}$ and $x=-2$
3. $x^4+4x^2-32=0$	$x=\pm 2$ and $x=\pm 2i\sqrt{2}$
4. $x^6-26x^3-27=0$	$x=3$ and $x=-1$
5. $2x^{\frac{2}{3}}+9x^{\frac{1}{3}}-5=0$	$x=\frac{1}{8}$ and $x=-125$
6. $x^{\frac{2}{5}}-3x^{\frac{1}{5}}-18=0$	$x=7{,}776$ and $x=-243$

GRAPHING A QUADRATIC FUNCTION OF THE FORM

$f(x) = ax^2 + bx + c \ (a \neq 0)$

The graph of $f(x) = ax^2 + bx + c$ is a parabola. Use the coefficients *a, b,* and *c* to find the following:

1. The **parabola opens upward** when $a > 0$. The **parabola opens downward** when $a < 0$.

2. The ***x*-coordinate of the vertex** of the parabola is $x = -\frac{b}{2a}$. The ***y*-coordinate of the vertex** is $f\left(-\frac{b}{2a}\right)$. The vertex is $\left(\frac{-b}{2a},\ f\left(\frac{-b}{2a}\right)\right)$.

3. The **axis of symmetry** is the vertical line passing through the vertex.

4. The ***y*-intercept** is determined by the value of $f(x)$ when $x = 0$: the *y*-intercept is $(0,\ f(0))$ or $(0, c)$.

5. The ***x*-intercepts (if any)** are determined by the values of *x* that make $f(x) = 0$. The *x*-intercept/s, if any, is $(x,\ 0)$. To find them, solve the quadratic equation $ax^2 + bx + c = 0$.

GRAPHING A QUADRATIC FUNCTION OF THE FORM

$f(x)=ax^2+bx+c \ (a\neq 0)$

Example: Graph $f(x)=2x^2-8x+6$

$a=2,\ b=-8,\ c=6$

Determine the values of *a, b,* and *c*.

1. $a=2$; parabola **opens upwards**

The parabola opens upward $(a>0)$.

2. $x=-\dfrac{-8}{2(2)}=2$

$f(2)=2(2)^2-8(2)+6$

$f(2)=-2$

The vertex is $\left(x, f\left(-\dfrac{b}{2a}\right)\right)=(2,-2)$

The *x*-coordinate of the vertex is $x=-\dfrac{b}{2a}$.

The *y*-coordinate of the vertex is $f\left(-\dfrac{b}{2a}\right)$.

3. $x=2$ **is the axis of symmetry.**

Axis of symmetry is the vertical line passing through the vertex.

4. $f(0)=2(0)^2-8(0)+6$

$f(0)=6$

The *y*-intercept is $(0,6)$

Let $x=0$.
The *y*-intercept is $(0, f(0))$.

5. $f(x)=2x^2-8x+6$

$0=2x^2-8x+6$

$0=2(x^2-4x+3)$

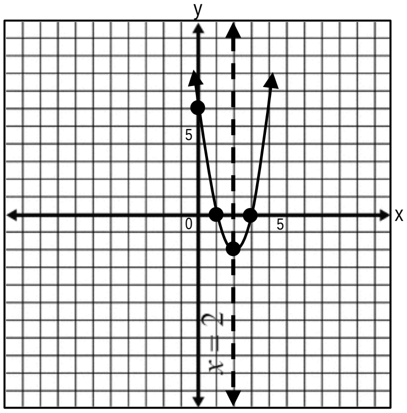
$0=2(x-3)(x-1)$

$x-3=0 \qquad x-1=0$

$x=3 \qquad\quad x=1$

Let $f(x)=0$. The *x*-intercept/s, if any, is $(x,0)$.

The *x*-intercepts are $(3,0)$ **and** $(1,0)$**.**



Notes

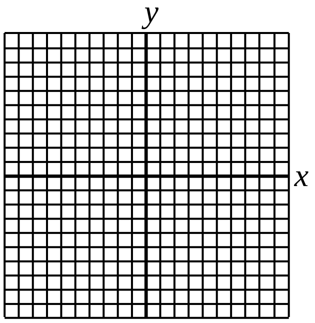
axis of symmetry: *x* = 2
y-intercept: (0,6)
x-intercepts: (1,0) and (3,0)

GRAPHING A QUADRATIC FUNCTION OF THE FORM
$f\left(x\right)=ax^2+bx+c\ (a\neq 0)$

Problems **Answers**

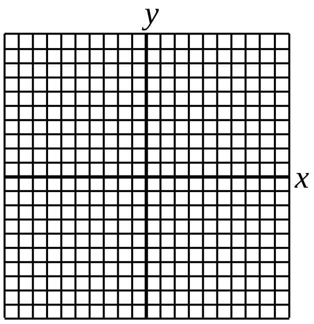
Use the 5-step process for graphing these quadratic functions.

1. $f\left(x\right)=2x^2+4x-6$



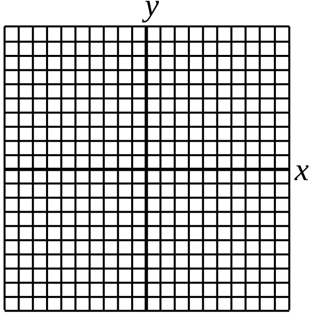
Step 1.Opens upward since 2 > 0.
Step 2. Vertex is $\left(-1,-8\right)$.
Step 3. Axis of symmetry is $x=-1$.
Step 4. *y*-intercept is $\left(0,-6\right)$.
Step 5. The *x*-intercepts are $\left(-3,0\right)$ and $\left(1,0\right)$.

2. $f\left(x\right)=x^2-2x-35$



Step 1. Opens upward since 1 > 0.
Step 2. Vertex is $\left(1,-36\right)$.
Step 3.Axis of symmetry is $x = 1$.
Step 4. *y*-intercept is $\left(0,-35\right)$.
Step 5. *x*-intercepts are $\left(7,0\right)$ and $\left(-5,0\right)$.

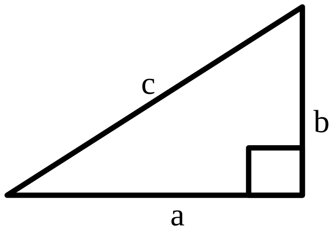
3. $f\left(x\right)=-x^2+4x-3$



Step 1. Opens downward since $-1<0$.
Step 2. Vertex is (2,1).
Step 3. Axis of symmetry is $x = 2$.
Step 4. *y*-intercept is $\left(0,-3\right)$.
Step 5. *x*-intercepts are (1,0) and (3,0).

PYTHAGOREAN THEOREM

In a right triangle the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the lengths of the legs (the sides that form the right angle). If *c* is the hypotenuse and *a* and *b* are the lengths of the legs, this property can be stated as:



$$c^2 = a^2 + b^2 \quad \text{or} \quad c = \sqrt{a^2 + b^2}$$

$$a^2 = c^2 - b^2 \quad \text{or} \quad a = \sqrt{c^2 - b^2}$$

$$b^2 = c^2 - a^2 \quad \text{or} \quad b = \sqrt{c^2 - a^2}$$

Examples: Find the length of the unknown side in the following right triangles.

1. Given $a = 3$ and $b = 5$, find *c*.

$c = \sqrt{9 + 25}$

$c = \sqrt{34}$

2. Given $b = 12$ and $c = 13$, find *a*.

$a = \sqrt{169 - 144}$

$a = \sqrt{25}$

$a = 5$

3. Given $c = 6$ and $a = \sqrt{5}$, find *b*.

$b = \sqrt{36 - 5}$

$b = \sqrt{31}$

PYTHAGOREAN THEOREM

Problems	Answers
Find the length of the unknown side in the following right triangles.	
1. a = 3 ft. b = 4 ft.	c = 5 ft.
2. b = 16 m. c = 20 m.	a = 12 m.
3. a = 6 yd. c = 10 yd.	b = 8 yd.
4. A 17-foot ladder is placed against the wall of a house. If the bottom of the ladder is 8 feet from the house, how far from the ground is the top of the ladder?	15 ft.
5. Find the diagonal of a rectangle whose length is 8 meters and whose width is 6 meters.	10 m.
6. Find the width of a rectangle whose diagonal is 13 feet and length is 12 feet.	5 ft.

EXPONENTIAL AND LOGARITHMIC FUNCTIONS

EXPONENTIAL FUNCTION

An **exponential function** is a function of the form $f(x) = b^x$ $\left(\text{or } y = b^x\right)$

where $b > 0, b \neq 1$ and x is a real number

note: b is the base and x is the exponent.

LOGARITHMIC FUNCTION

If $b > 0$, $b \neq 1$, and $x > 0$, then $\log_b y = x$

$\log_b y = x$ **is equivalent to** $y = b^x$

note: b is the base and x is the exponent.

Example: Write $\log_2 32 = 5$ in exponential form.

$$\underset{\substack{\nearrow \\ \text{base}}}{\log_{2}} 32 = \underset{\substack{\nwarrow \\ \text{exponent}}}{5}$$

Answer: $2^5 = 32$ is in exponential form.

Example: Write $4^3 = 64$ in logarithmic form.

$$4^3 = 64$$

Answer: $\log_4 64 = 3$ is in logarithmic form.

EXPONENTIAL AND LOGARITHMIC FUNCTIONS

PROPERTIES OF LOGARITHMS

$\log_b 1 = 0$

$\log_b b = 1$

$\log_b b^x = x$

$b^{\log_b x} = x$

Examples:

Evaluate: $\log_6 1$

Answer: $\mathbf{\log_6 1 = 0}$ Use the $\log_b 1 = 0$ property.

Evaluate: $\log_3 3$

Answer: $\mathbf{\log_3 3 = 1}$ Use the $\log_b b = 1$ property.

Evaluate: $\log_8 64$

$\log_8 64 = x$

$\log_8 8^2 = x$ Notice that $64 = 8^2$ and now the bases are the same.

$2 = x$ Use $\log_b b^x = x$ property.

Answer: $\mathbf{\log_8 64 = 2}$

Evaluate: $6^{\log_6 24}$

$6^{\log_6 24} = x$ Notice that the bases are the same.

$24 = x$ Use $b^{\log_b x} = x$ property.

Answer: $\mathbf{6^{\log_6 24} = 24}$

EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Product Rule for Logarithms

$$\log_b MN = \log_b M + \log_b N$$

Quotient Rule for Logarithms

$$\log_b \frac{M}{N} = \log_b M - \log_b N$$

Power Rule for Logarithms

$$\log_b M^n = n \cdot \log_b M$$

Examples:

Expand: $\log_6 5r$

$\log_6 5r = \log_6 5 + \log_6 r$ Use the Product Rule.

Expand: $\log_7 \frac{12}{c}$

$\log_7 \frac{12}{c} = \log_7 12 - \log_7 c$ Use the Quotient Rule.

Expand: $\log_4 \sqrt[3]{25}$

$\log_4 \sqrt[3]{25} = \log_4 25^{\frac{1}{3}}$ Recall $\sqrt[3]{25}$ is $25^{\frac{1}{3}}$.

$= \frac{1}{3}\log_4 25$ Use the Power Rule.

Write as a single logarithm: $3\log_4 x - \log_4 y$

$3\log_4 x - \log_4 y = \log_4 x^3 - \log_4 y$ Use the Power Rule.

$= \log_4 \frac{x^3}{y}$ Since the bases are the same, use the Quotient Rule

EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Problems	**Answers**
1. Write $\log_5 125 = 3$ in exponential form	$5^3 = 125$
2. Write $2^{-3} = \frac{1}{8}$ in logarithmic form	$\log_2 \frac{1}{8} = -3$
3. Evaluate: $\log_3 9$	2
4. Evaluate: $\log 1000$ (When a base is not indicated, it is a **common log** with a base of 10.)	3
5. Expand: $\log_b \frac{x^2 y^3}{z}$	$2\log_b x + 3\log_b y - \log_b z$
6. Write as a single logarithm: $4\log_3 x + \log_3 (x+2) - \log_3 8$	$\log_3 \frac{x^4 \left(x+2\right)}{8}$

VOCABULARY USED IN APPLICATION PROBLEMS

Addition

- add
- sum
- total
- plus
- in all
- more than
- together
- increased by
- all together
- combined

English	**Algebra**
The **sum** of a number and 4.	$x+4$
Seven **more than** a number.	$x+7$
Six **increased by** a number.	$6+x$
A number **added to** 8.	$8+x$
A number **plus** 4.	$x+4$

Subtraction

- subtracted from
- difference
- take away
- less than
- minus
- remain
- decreased by
- have left
- are left
- more
- fewer

English	**Algebra**
The **difference** of a number and 3.	$x-3$
The **difference** of 3 and a number.	$3-x$
Five **less than** a number.	$x-5$
A number **decreased by** 3.	$x-3$
A number **subtracted from** 8.	$8-x$
Eight **subtracted from** a number.	$x-8$
Two **minus** a number.	$2-x$

Be careful with subtraction. The order is important. Three less than a number is $x-3$.

VOCABULARY USED IN APPLICATION PROBLEMS

Multiplication

- product of
- times
- multiplied by
- of

English	Algebra
The **product** of 3 and a number.	$3x$
Three-fourths **of** a number.	$\frac{3}{4}x$
Four **times** a number.	$4x$
A number **multiplied** by 6.	$6x$
Double a number.	$2x$
Twice a number.	$2x$

Division

- divided by
- quotient of
- separated into equal parts
- shared equally

English	Algebra
The **quotient** of a number and 3.	$x \div 3$ or $\frac{x}{3}$
The **quotient** of 3 and a number.	$3 \div x$ or $\frac{3}{x}$
A number **divided by** 6.	$x \div 6$ or $\frac{x}{6}$
Six **divided by** a number.	$6 \div x$ or $\frac{6}{x}$

Be careful with division. The order is important. A number divided by 6 is $\frac{x}{6}$

REVIEW OF INTERMEDIATE ALGEBRA

1 – 7: Completely factor each expression.

1. $6x^3 - 15x^2 - 9x$
2. $-2x^5 + 250x^2$
3. $16x^8 - 81y^4$
4. $\left(a+b\right)^2 + 3\left(a+b\right) - 4$
5. $54x^3 - 16y^3$
6. $-3t^5 - 24k^3t^2$
7. $64z^3 + 1$

8 – 13: Perform the operations, simplify if possible.

8. Reduce: $\dfrac{2x^2-8}{3x^2-5x-2}$

9. $\dfrac{2x^2-9x+9}{2x^2-x-3} \bullet \dfrac{4x^2+3x-1}{x^2-3x}$

10. $\dfrac{x^2-2x-35}{3x^2+27x} \div \dfrac{x^2+7x+10}{6x^2+12x}$

11. $\dfrac{3}{x^2+x-6} + \dfrac{1}{x^2+3x-10}$

12. $\dfrac{\dfrac{x^2-16}{5x+10}}{\dfrac{2x-8}{25}}$

13. $\dfrac{x}{x-1}-\dfrac{x-6}{x-4}$

14 – 18: Write the equation of the indicated lines. State your answers in slope intercept form.

14. Write the equation of the line through (–4,2) with a slope of ⅔.

15. Write the equation of the line through (1,5) and (0,–2).

16. Write the equation of the line going through (3,–1) and (–6,2).

17. Write the equation of the line going through (0,–5) and:
    a) parallel to $2y+x=6$
    b) perpendicular to $2y+x=6$

18. Write the equation of the line going through (1,–3) and:
    a) parallel to $y=3x-8$
    b) perpendicular to $y=3x-8$

19 – 29: Simplify. Assume all variables represent positive numbers.

19. $\sqrt{250x^3y^5}$

20. $\sqrt[3]{-64x^3y^8}$

21. $(36)^{\frac{-3}{2}}$

22. $\dfrac{t^{-\frac{1}{2}}t^{\frac{2}{3}}}{t^{\frac{3}{2}}}$

REVIEW OF INTERMEDIATE ALGEBRA

23. $\left(8x^{\frac{1}{2}}y^{-1}\right)^{\frac{-2}{3}}$

24. $\left(x^{\frac{2}{3}}y^{\frac{1}{2}}\right)\left(x^{\frac{3}{4}}y^{\frac{1}{3}}\right)$

25. $\sqrt[4]{9x^3y}\bullet\sqrt[4]{2xy^5}$

26. $\sqrt{-9}\sqrt{100}$

27. $\dfrac{-\sqrt{-225}}{\sqrt{-16}}$

28. $\dfrac{5i^3}{2\sqrt{-4}}$

29. $\dfrac{3i}{6-i}$ (Write answer in the $a+bi$ form.)

30. Given: $f\left(x\right)=2x^2-8x+6$

a) Identify the *x* intercepts as points
b) Identify the *y* intercept as a point
c) Identify the vertex
d) Identify the domain using interval notation
e) Identify the range using interval notation
f) Sketch a graph of $f\left(x\right)$.

REVIEW OF INTERMEDIATE ALGEBRA

31– 42: Solve for *x*.

31. $x - 7\sqrt{x} + 10 = 0$

32. $\sqrt{3x+4} + x = 8$

33. $(x+1)^2 = 54$

34. $4x^2 + 6x + 1 = 0$

35. $\frac{1}{a} + \frac{1}{c} = \frac{1}{t}$; solve for *a*

36. $\sqrt{3a+1} = a - 1$

37. $\frac{1}{2}(x-6) = \frac{3}{4}x + 9$

38. $\frac{3}{x-2} + \frac{1}{x} = \frac{2(3x+2)}{x^2 - 2x}$

39. $2 + \sqrt{x} = \sqrt{2x+7}$

40. $3x^2 + 4x + 2 = 0$

41. $\frac{x^2}{3} - x - \frac{1}{6} = 0$

42. $3x^2 = 4x - 6$

43. Given $f(x) = 3x^2 - 2x + 1$ and $g(x) = x - 2$ find:

a) $f(-1)$

b) $(f+g)(x)$

c) $(f-g)(x)$

d) $(f \circ g)(x)$

e) $\left(\frac{f}{g}\right)(-2)$

44. For each function

1) Identify the type of function

2) Graph the function

3) State the domain and range

a) $f(x) = -\frac{2}{3}x + 4$

b) $f(x) = |x - 3|$

c) $f(x) = \sqrt{x} + 2$

d) $f(x) = x^2 - 2x - 3$

e) $f(x) = 2^x + 1$

45. Identify the domain using interval notation.

a) $f(x) = 3x + 5$

b) $f(x) = x^2 - 4x - 5$

c) $f(x) = \sqrt{x+1}$

d) $f(x) = \frac{x}{x^2 + 3x - 4}$

e) $f(x) = \log_3(x+2)$

REVIEW OF INTERMEDIATE ALGEBRA

46 – 48: Solve and state the solution using interval notation:

46. $x^2+3x-54>0$

47. $\dfrac{x-8}{x+2}\le 0$

48. $x^2-3x\le -2$

49. Write as an exponential equation $\log_2 8=3$

50. Write as a logarithmic equation $4^0=1$

51. Evaluate: $\log_6 36$

52. Solve for *x*: $\log_3 x=-4$

53. Use logarithmic properties to rewrite:

a) $\log_3 x\sqrt[4]{y}$ b) $\log\dfrac{x^2w}{y^3}$

54. Write the expression as a single logarithm.

$$\log_2(x-2)-\log_2 x-3\log_2 y$$

56 – 67: Solve the following word problems. Show all steps including identifying the unknown, writing an equation, solving the equation and answering in words.

55. The length of a rectangular carpet is 4 feet greater than twice its width. If the area is 48 square feet, find the carpet's length and width.

56. The length of a rectangular sign is 3 feet longer than the width. If the sign has space for 54 square feet of advertising, find its length and its width.

57. You can design a website in 30 hours. Your friend can design the same site in 20 hours. How long will it take to design the website if you both work together?

58. You can travel 40 miles on a motorcycle in the same time that it takes to travel 15 miles on a bicycle. If your motorcycle's rate is 20 miles per hour faster than your bicycle's, find the average rate for each.

59. The function $f(x) = -x^2 + 46x - 360$ models the daily profit, $f(x)$, in hundreds of dollars, for a company that manufactures x computers daily. How many computers should be manufactured each day to maximize the profit? What is the maximum daily profit?

60. A homeowner estimates that it will take him 7 days to roof his house. A professional roofer estimates that he can roof the house in 4 days. How long will it take if the homeowner helps the roofer?

61. A flare is fired directly upward into the air from a boat that is experiencing engine problems. The height of the flare (in feet) above the water, *t* seconds after being fired, is given by the formula $h = -16t^2 + 112t + 15$. If the flare is designed to explode when it reaches its highest point, at what height will this occur?

62. A man rollerblades at a rate of 6 miles per hour faster than he jogs. In the same time it takes him to rollerblade 5 miles, he can jog 2 miles. How fast does he jog?

63. An amount of money invested in a savings account for one year earns $100 in interest. The same amount invested in a CD for one year earns $120 because the rate is 1% more. Find the rate for the savings account.

64. A hot-air balloonist accidentally dropped his camera overboard while traveling at a height of 1,600 feet. The height *h*, in feet, of the camera *t* seconds after being dropped is given by $h = -16t^2 + 1600$. In how many seconds will the camera hit the ground?

65. One leg of a right triangle is 2 feet less than the hypotenuse. The second leg is 4 feet less than the hypotenuse. Find the length of all 3 sides.

66. The yearly budget B (in billions of dollars) for NASA is approximated by the quadratic equation $B = 0.0596x^2 - .3811x + 14.2709$ where *x* is the number of years since 1995 and $0 \le x \le 9$. In what year does the model indicate that NASA's budget was $15 billion? Round to the nearest year.

Review Answers

1. $3x(2x+1)(x-3)$

2. $-2x^2(x-5)(x^2+5x+25)$

3. $(4x^4+9y^2)(2x^2+3y)(2x^2-3y)$

4. $(a+b+4)(a+b-1)$

5. $2(3x-2y)(9x^2+6xy+4y^2)$

6. $-3t^2(t+2k)(t^2-2tk+4k^2)$

7. $(4z+1)(16z^2-4z+1)$

8. $\dfrac{2(x+2)}{3x+1}$

9. $\dfrac{4x-1}{x}$

10. $\dfrac{2(x-7)}{x+9}$

11. $\dfrac{4x+18}{(x+3)(x-2)(x+5)}$

12. $\dfrac{5(x+4)}{2(x+2)}$

13. $\dfrac{3x-6}{(x-1)(x-4)}$

14. $y=\dfrac{2}{3}x+\dfrac{14}{3}$

15. $y=7x-2$

REVIEW OF INTERMEDIATE ALGEBRA

16. $y=-\frac{1}{3}x$

17. a) $y=-\frac{1}{2}x-5$ b) $y=2x-5$

18. a) $y=3x-6$ b) $y=\frac{-1}{3}x-\frac{8}{3}$

19. $5xy^2\sqrt{10xy}$

20. $-4xy^2\sqrt[3]{y^2}$

21. $\frac{1}{216}$

22. $\frac{1}{t^{\frac{4}{3}}}$

23. $\frac{y^{\frac{2}{3}}}{4x^{\frac{1}{3}}}$

24. $x^{\frac{17}{12}}y^{\frac{5}{6}}$

25. $xy\sqrt[4]{18y^2}$

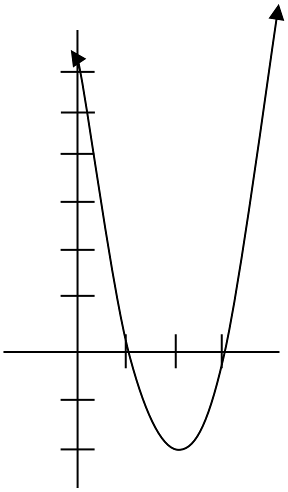
26. $30i$

27. $-\frac{15}{4}$

28. $\frac{-5}{4}$

29. $-\frac{3}{37}+\frac{18}{37}i$

30. a) $(1,0)(3,0)$
    b) $(0,6)$
    c) $(2,-2)$
    d) $(-\infty,\infty)$
    e) $[-2,\infty)$
    f)



REVIEW OF INTERMEDIATE ALGEBRA

31. $x = 4,\ x = 25$

32. $x = 4,\ x = 15$ is an extraneous solution.

33. $x = -1 \pm 3\sqrt{6}$

34. $x = \dfrac{-3 \pm \sqrt{5}}{4}$

35. $a = \dfrac{ct}{c - t}$

36. $a = 5$ $\left(a = 0 \text{ extraneous solution}\right)$

37. $x = -48$

38. $x = -3$

39. $x = 1,\ x = 9$

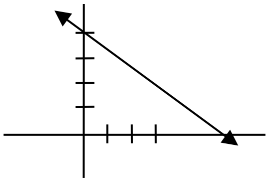
40. $x = \dfrac{-2 \pm i\sqrt{2}}{3}$ or $x = \dfrac{-2}{3} \pm \dfrac{i\sqrt{2}}{3}$

41. $x = \dfrac{3 \pm \sqrt{11}}{2}$

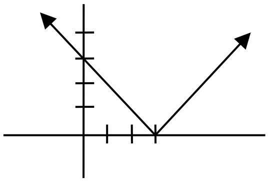
42. $x = \dfrac{2 \pm i\sqrt{14}}{3}$ or $x = \dfrac{2}{3} \pm \dfrac{i\sqrt{14}}{3}$

43. a) 6
    b) $3x^2 - x - 1$
    c) $3x^2 - 3x + 3$
    d) $3x^2 - 14x + 17$
    e) $-\dfrac{17}{4}$

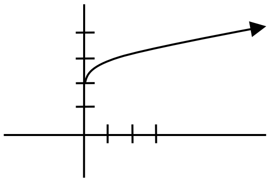
44. a) linear
    domain: $(-\infty, \infty)$
    range: $(-\infty, \infty)$



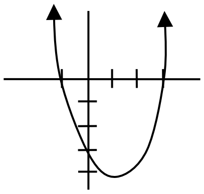
b) absolute value
    domain: $(-\infty, \infty)$
    range: $[0, \infty)$



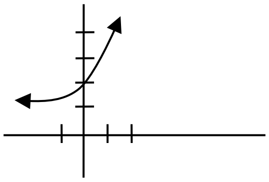
c) radical (square root)
domain: $[0,\infty)$
range: $[2,\infty)$



d) quadratic
domain: $(-\infty,\infty)$
range: $[-4,\infty)$



e) exponential
domain: $(-\infty,\infty)$
range: $(1,\infty)$



45. a) $(-\infty,\infty)$
    b) $(-\infty,\infty)$
    c) $[-1,\infty)$
    d) $(-\infty,-4)\cup(-4,1)\cup(1,\infty)$
    e) $(-2,\infty)$

46. $(-\infty,-9)\cup(6,\infty)$

47. $(-2,8]$

48. $[1,2]$

49. $2^3=8$

50. $\log_4 1=0$

51. 2

52. $x=\dfrac{1}{81}$

53. a) $\log_3 x + \frac{1}{4}\log_3 y$

    b) $2\log x + \log w - 3\log y$

54. $\log_2 \frac{(x-2)}{xy^3}$

55. The width is 4 feet and the length is 12 feet.

56. The width is 6 feet and the length is 9 feet.

57. It will take 12 hours.

58. The bike goes 12 mph, and the motorcycle goes 32 mph.

59. Sell 23 computers for a maximum profit of $16,900.

60. It will take them $2\frac{6}{11}$ days.

61. It will explode at 211 feet.

62. He jogs 4 mph.

63. The rate is 5%.

64. In 10 seconds, the camera will hit the ground.

65. The sides are 6 feet, 8 feet, and 10 feet.

66. In 2003, NASA’s budget was $15 billion.