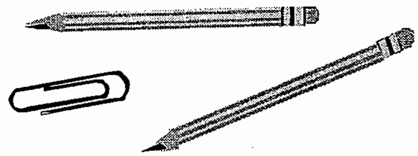
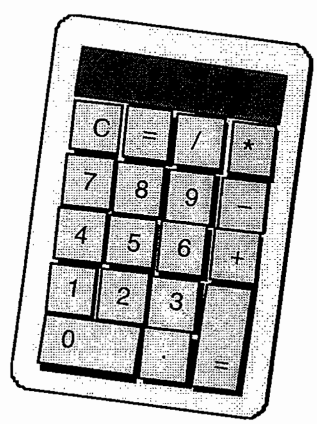


COLLEGE ALGEBRA AND TRIGONOMETRY

REVIEW



This manual is designed as a brief review of college algebra and trigonometry skills. It includes a review of some important concepts, as well as sample worksheets. This material **DOES NOT** reflect the content of the assessment test; it is **ONLY** a review.

The mathematics assessment test is a diagnostic tool to assist the college in determining the appropriate placement for you to be successful in the course work you will be taking, not only at McHenry County College, but at any other schools you may choose to attend in the future.

Thorough understanding of the review materials will help prepare you for the assessment test. However, this **DOES NOT** guarantee a passing score on the test. This material should not serve as a teaching tool, but rather as brief review of skills covered in courses already taken. Four-function calculators will be provided during the assessment test. Please keep these facts in mind while reviewing the materials.

In addition to this manual, you may wish to review arithmetic and algebra textbooks, as well as video tapes, all of which are available in the Leaning Resource Center (library). It is essential that you prepare for the assessment test in a serious manner. The test may be taken a maximum of two times. The results of the assessment test will determine your mathematics placement at McHenry County College.

Part 1

COLLEGE ALGEBRA

To solve non-linear Inequalities:

1. Locate critical values.

   - In polynomials critical values are the zeros
   - In rational expressions critical values are zeros of the numerator and the denominator.

2. Use the critical values to divide the number line into regions.

3. Test a value in each region.

4. Test critical values to see if they are part of the solution set.

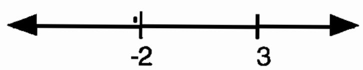
Note: The above steps should only be used after one side is zero.

Examples: 1. Solve $x(x - 1) \geq 6$

$x(x - 1) - 6 \geq 0$

$x^2 - x - 6 \geq 0$

$(x - 3)(x + 2) \geq 0$ Critical values are x = -2, 3



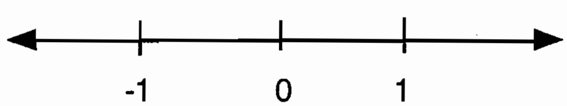
T.V. =-3 T.V. = 0 T.V. = 4

True False True Solution: $(-\infty, -2] \cup [3, \infty)$

2. Solve $\frac{1}{x} \geq x$

$\frac{1}{x} - x \geq 0$

$\frac{1-x^2}{x} \geq 0$ Critcal values are x = -1, 0, 1



T.V. = -2 | T.V. =-.5 | T. V. =.5 | T.V. = 2

True | False | True | False Solution: $(-\infty, -1] \cup (0, 1]$

College Algebra/Trig. Review **Non-linear Inequalities**

Solve the following inequalities. Put answers in interval form.

1.	$x^3(x-2)(x+4) < 0$	1.	$(-\infty,-4)\cup(0,2)$
2.	$x^2 + x - 6 \geq 0$	2.	$(-\infty,-3]\cup[2,\infty)$
3.	$x(x+5) > 6$	3.	$(-\infty,-6)\cup(1,\infty)$
4.	$3x^2 + 4x < x^2 + x + 2$	4.	$\left(-2,\frac{1}{2}\right)$
5.	$\frac{x-5}{2x+7} \geq 0$	5.	$\left(-\infty,\frac{-7}{2}\right)\cup[5,\infty)$
6.	$\frac{3+x}{x-2} < 6$	6.	$(-\infty,2)\cup(3,\infty)$
7.	$\frac{1}{x} \leq \frac{2}{x-2}$	7.	$[-2,0)\cup(2,\infty)$

- If f(x) and g(x) are two functions, then:
  1. $(f + g)(x) = f(x) + g(x)$
  2. $(f - g)(x) = f(x) - g(x)$
  3. $(fg)(x) = f(x)\,g(x)$
  4. $(f/g)(x) = \dfrac{f(x)}{g(x)}$ if $g(x) \neq 0$
  5. $(f o g)(x) = f(g(x))$
  6. $(g o f)(x) = g(f(x))$

- The difference quotient of a function f(x) is $\dfrac{f(x+h)-f(x)}{h}$

Examples: If $f(x) = x^2 + 2x$ and $g(x) = x + 1$

1. $(f + g)(x) = (x^2 + 2x) + (x + 1)$

   $= x^2 + 3x + 1$

2. $(f - g)(x) = x^2 + 2x - x - 1$

   $= x^2 + x - 1$

3. $(fg)(x) = (x^2 + 2x)(x + 1)$

   $= x^3 + 3x^2 + 2x$

4. $\left(\dfrac{f}{g}\right)(x) = \dfrac{(x^2 + 2x)}{(x+1)}$ *if* $x \neq -1$

5. $(f o g)(x) = f(x + 1)$

   $= (x + 1)^2 + 2(x + 1)$

   $= (x^2 + 2x + 1) + (2x + 2)$

   $= x^2 + 4x + 3$

6. $(g o f)(x) = g(x^2 + 2x)$

   $= x^2 + 2x + 1$

7. $$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + 2(x+h) - x^2 - 2x}{h}$$

   $$= \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h}$$

   $$= \frac{h(2x + h + 2)}{h}$$

   $$= 2x + h + 2$$

College Algebra/Trig. Review **Combinations of Functions**

If $f(x) = x^2 - 3x$, $g(x) = x + 2$ and $p(x) = \frac{1}{x}$ find:

1.	$(f + g)(x)$	1.	$x^2 - 2x + 2$
2.	$(f\,g)(x)$	2.	$x^3 - x^2 - 6x$
3.	$\left(\frac{f}{g}\right)(x)$	3.	$\frac{x^2-3x}{x+2}$ $x \neq -2$
4.	$(f - g)(x)$	4.	$x^2 - 4x - 2$
5.	$(f \circ g)(x)$	5.	$x^2 + x - 2$
6.	$(g \circ f)(x)$	6.	$x^2 - 3x + 2$
7.	$(p \circ g)(x)$	7.	$\frac{1}{x+2}$ $x \neq -2$
8.	$(g \circ f \circ p)(x)$	8.	$\frac{1}{x^2} - \frac{3}{x} + 2$
9.	$f(x + h)$	9.	$x^2 + 2xh + h^2 - 3x - 3h$
10.	$\frac{f(x+h)-f(x)}{h}$	10.	$2x + h - 3$

- If f(x) and g(x) are inverse functions then

$$f(g(x)) = g(f(x)) = x$$

- If f(x) has an inverse then f(x) must be a one to one function.
- The graphs of inverse functions are reflections over the line y = x.
- To find the inverse:
  1. Switch x and y
  2. Solve for y

Example: Find the inverse of $f(x) = \frac{3}{2x-1}$

$$y = \frac{3}{2x-1}$$

$$x = \frac{3}{2y-1} \qquad \text{Switch } x \text{ and } y$$

$$x(2y-1) = 3 \qquad \text{Solve for } y$$

$$2xy - x = 3$$

$$2xy = 3 + x$$

$$y = \frac{3+x}{2x}$$

$$f^{-1}(x) = \frac{3+x}{2x}$$

Check: $f(f^{-1}(x)) = f\left(\frac{3+x}{2x}\right)$

$$= \frac{3}{2\left(\frac{3+x}{2x}\right)-1}$$

$$= \frac{3}{\frac{3+x}{x} - \frac{x}{x}}$$

$$= x$$

It can also be shown that $f^{-1}(f(x)) = x$

College Algebra/Trig. Review **Inverse Functions**

Find $f^{-1}(x)$ for the following:

1. $f(x) = 2x + 3$

2. $f(x) = x^3 - 1$

3. $f(x) = \frac{4}{x}$

4. $f(x) = \frac{4}{x+2}$

5. $f(x) = (x-1)^2, \ x \geq 1$

6. $f(x) = \frac{2x+1}{x-1}$

7. $f(x) = \frac{3x+1}{x}$

Answers:

1. $f^{-1}(x) = \frac{1}{2}(x-3)$

2. $f^{-1}(x) = \sqrt[3]{x+1}$

3. $f^{-1}(x) = \frac{4}{x}$

4. $f^{-1}(x) = \frac{4}{x} - 2$

5. $f^{-1}(x) = \sqrt{x} + 1$

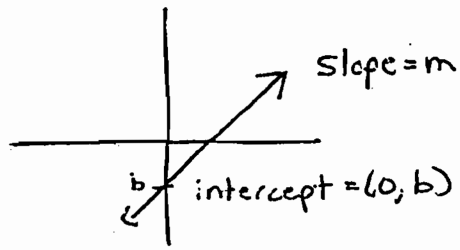
6. $f^{-1}(x) = \frac{x+1}{x-2}$

7. $f^{-1}(x) = \frac{1}{x-3}$

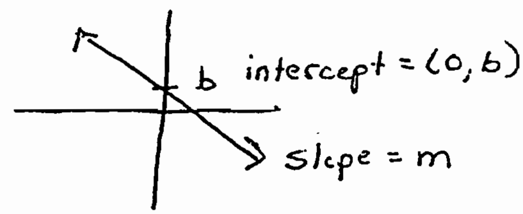
College Algebra/Trig. Review **Important Graphs**

The following basic graphs should be memorized.

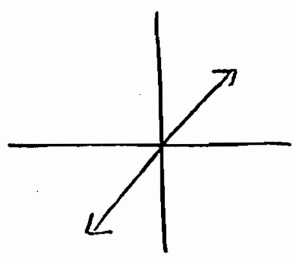
1. $f(x) = mx + b \quad m > o$



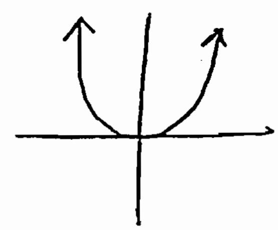
2. $f(x) = mx + b \quad m < o$



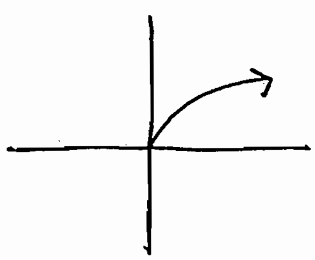
3. $f(x) = x$



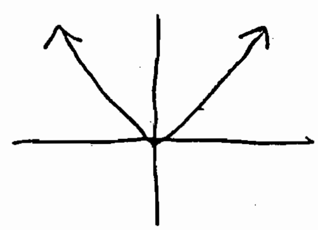
4. $f(x) = x^2$



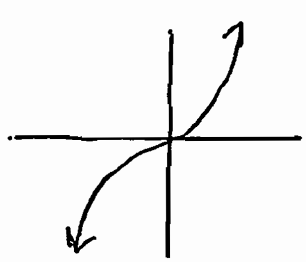
5. $f(x) = \sqrt{x}$



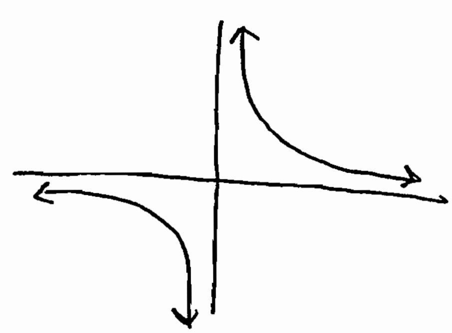
6. $f(x) = |x|$



7. $f(x) = x^3$



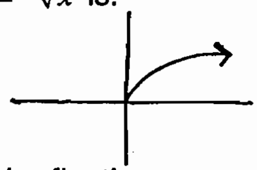
7. $f(x) = \frac{1}{x}$



Given the graph of a function f(x):

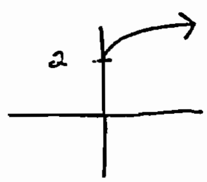
f(x)+h moves the graph up h units
f(x)-h moves the graph down h units
f(x+h) moves the graph left h units
f(x-h) moves the graph right h units
f(-x) reflects the graph over the y axis
-f(x) reflects the graph over the x axis

Example: The graph of f(x) = $\sqrt{x}$ is:

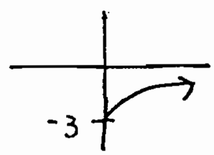


Using shifts, translations and reflections we get:

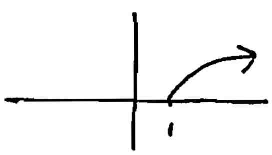
1. f(x) + 2 = $\sqrt{x}$ + 2



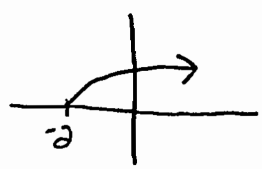
2. f(x) -3 = $\sqrt{x}$ -3



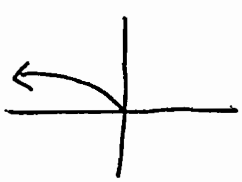
3. f(x - 1) = $\sqrt{x-1}$



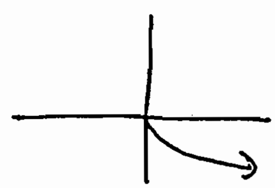
4. f(x + 2) = $\sqrt{x+2}$



5. f(-x) = $\sqrt{-x}$



6. -f(x) = $-\sqrt{x}$



Graph the following using shifts/translations and reflections.

1. $f(x) = x^2 + 2$

2. $f(x) = (x-3)^3$

3. $f(x) = -|x|$

4. $f(x) = \sqrt{x} - 1$

5. $f(x) = (x-1)^2 + 2$

6. $f(x) = -\sqrt{x+2}$

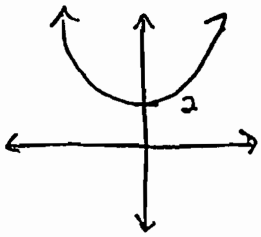
7. $f(x) = -(x+2)^3 - 1$

8. $f(x) = |x-1| + 4$

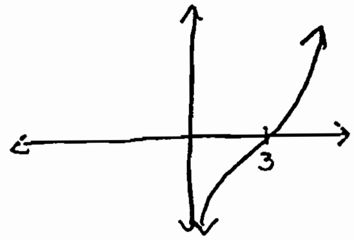
College Algebra/Trig. Review **Shifts/Translations/Reflections**

Solutions to graphs:

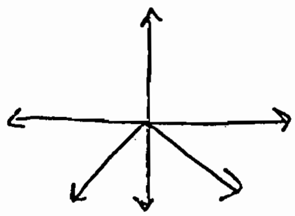
1. $f(x) = x^2 + 2$
moved up 2



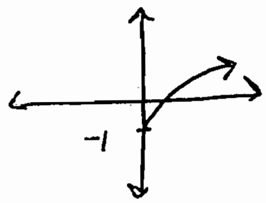
2. $f(x) = (x - 3)^3$
moved right 3



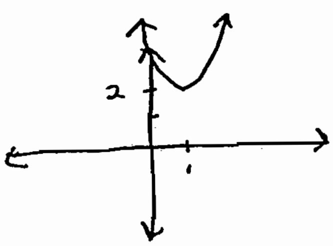
3. $f(x) = -|x|$
reflects over x axis



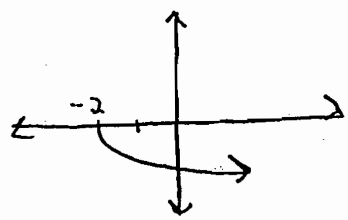
4. $f(x) = \sqrt{x} - 1$
moved down 1



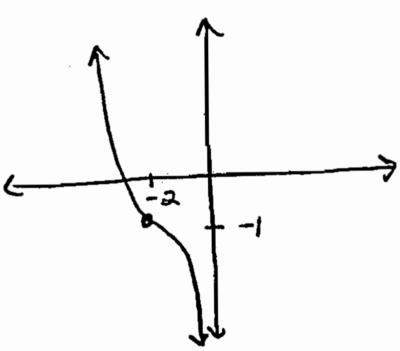
5. $f(x) = (x - 1)^2 + 2$
moved 1 right and
2 up



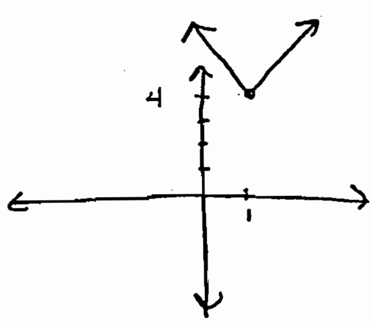
6. $f(x) = -\sqrt{x + 2}$
moved 2 left and
reflects over x axis



7. $f(x) = -(x + 2)^3 - 1$
moved 2 left, down 1
and reflected over x axis



8. $f(x) = |x - 1| + 4$
moved 1 right and
4 up



- Quadratic functions are functions of the form $f(x) = ax^2 + bx + c$
- Standard form is $f(x) = a(x - h)^2 + k$ or equivalently $f(x) - k = a(x - h)^2$
- A quadratic function can be placed in standard form by completing the square.
- The graph of a quadratic function is a parabola with a vertex of (h,k) and opens up if $a > 0$ and down if $a < 0$.
- The vertex is a maximum if $a < 0$.
- The vertex is a minimum if $a > 0$.

Examples: 1. Find the vertex of $f(x) = 3x^2 - 6x + 2$

$f(x) = 3(x^2 - 2x \quad) + 2$

$= 3(x^2 - 2x + 1) + 2 - 3$ Complete the square

$= 3(x - 1)^2 - 1$

Vertex is (1, - 1)

2. Find the minimum or the maximum value of $f(x) = -2x^2 - 8x + 3$

$f(x) = -2(x^2 + 4x \quad) + 3$ The value is a max since $a < 0$

$= -2(x^2 + 4x + 4) + 3 + 8$ Complete the square

$= -2(x + 2)^2 + 11$

Vertex is (-2, 11)

Maximum value is 11

Put the following in standard form and find the vertex.

1.	$f(x) = x^2 + 2x - 5$	1.	S.F.: $f(x) = (x+1)^2 - 6$ V.: $(-1, -6)$
2.	$f(x) = 2x^2 + 4x - 8$	2.	S.F.: $f(x) = 2(x+1)^2 - 10$ V.: $(-1, -10)$
3.	$f(x) = -3x^2 + 6x - 2$	3.	S.F.: $f(x) = -3(x-1)^2 + 1$ V: $(1, 1)$
4.	$f(x) = x^2 + 3x + 2$	4.	S.F.: $f(x) = (x+\frac{3}{2})^2 - \frac{1}{4}$ V: $(-\frac{3}{2}, -\frac{1}{4})$
5.	$f(x) = 2x^2 - 5x + 4$	5.	S. F.: $f(x) = 2(x-\frac{5}{4})^2 + \frac{7}{8}$ V.: $(\frac{5}{4}, \frac{7}{8})$
6.	Determine the maximum or minimum value of $f(x) = 2x^2 + 8x - 3$	6.	Minimum value of -11
7.	Determine the maximum or minimum value of $f(x) = -3x^2 + 12x - 2$	7.	Maximum value of 10

- Polynomial functions are of the form f(x) = $a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_o$
- The zeros of a polynomial function are the same as the roots or solutions to f(x) = 0. They are also the x coordinates of the x intercepts and (x - the zero) is a factor of f(x).
- A number c is a zero of f(x) if when f(x) is divided by x - c the remainder is zero.
- Possible zeros can be found using the Rational Root Theorem which says:

  The possible rational zeros to f(x) are of the form $\frac{p}{q}$ where p are the factors of a_o and q are the factors of a_n.

- Once a zero is found, the new function (bottom row of synthetic division) should be used. Check to see if this can be solved using other methods before continuing. These methods include factoring by grouping and the quadratic formula.

Example: f(x) = $2x^3 - 5x^2 - 4x + 3$

Possible rational zeros: $\pm\left(\frac{1,3}{1,2}\right) or \pm\left(1,3,\frac{1}{2},\frac{3}{2}\right)$

To find zero:

	2	-5	-4	3
		-2	7	-3
-1	2	-7	3	0

x = -1 is a zero

New equation: $2x^2 - 7x + 3$

(2x -1) (x - 3) other zeros $x = \frac{1}{2}, x = 3$

zeros: $x = -1, \frac{1}{2}, 3$

intercepts: (-1, 0) $(\frac{1}{2}, 0)$ (3, 0)

factored: (x + 1) (2x -1) (x -3)

solutions or roots to $2x^3 - 5x^2 - 4x + 3 = 0$: $x = -1, \frac{1}{2}, 3$

College Algebra/Trig. Review **Polynomial Functions and Equations**

1. Find all zeros for
$P(x) = x^3 - 7x - 6$

1. $x = 3, -2, -1$

2. Factor $P(x) = 6x^3 + x^2 - 19x + 6$

2. $(2x - 3)(x + 2)(3x - 1)$

3. Solve $x^4 + 3x^3 - 9x^2 - 12x + 20 = 0$

3. $x = 2, -2, \frac{-3 \pm \sqrt{29}}{2}$

4. Find all x intercepts for
$P(x) = 2x^3 + 7x^2 + x - 10$

4. $(1,0)$, $(-2,0)$, $(\frac{-5}{2}, 0)$

5. Find all roots to
$x^4 + 2x^3 + 3x^2 + 2x - 8 = 0$

5. $x = 1, -2, \frac{-1 \pm i\sqrt{15}}{2}$

6. Find a polynomial with integer coeficients and zeros of $x = \frac{1}{2}, \frac{2}{3}$ *and* -3.

6. $P(x) = 6x^3 + 11x^2 - 19x + 6$

7. Find a polynomial with integer coeficients and zeros of $x = 2 + i$, 3 and -3

7. $P(x) = x^4 - 4x^3 - 4x^2 + 36x - 45$

Rational functions are functions of the form:

$$R(x) = \frac{P(x)}{Q(x)}$$

where P(x) and Q(x) are polynomials.

Rational functions can be graphed using the following steps.

1. Set y = 0 and find x - intercepts (if any). Plot these points.
2. Set x = 0 and find the y - intercept (if it exists). Plot this point.
3. Locate any vertical asymptotes by setting the denominator to 0 and solve for x. Graph these lines, using a dotted line.
4. Locate any horizontal asymptotes by checking the degrees of the polynomials in the numerator and denominator. Graph using a dotted line. If:

   a. the degree of the numerator is less than the degree of the denominator, the asymptote is y = 0.
   b. the degrees are equal, the line is y = p/q where p and q are the leading coefficients of the two polynomials.

   Ex. $\frac{2x+1}{3x+2}$ Each polynomial has degree 1. Asymptote is y = 2/3.

   c. the degree of the numerator is one more than the degree of the denominator, divide the denominator into the numerator (the quotient, minus the remainder, will give you the slant asymptote.) Note: In this case there is no horizontal asymptote.

   Ex. $\frac{2x^2-2}{2x+8}$ slant asymptote is y = x - 4

   Divide:
$$\begin{array}{r} x - 4 \\ 2x+8 \overline{)\,2x^2 \qquad - 2} \\ \underline{2x^2 + 8x} \\ -8x - 2 \\ \underline{-8x - 32} \\ 30 \end{array}$$

NOTE: The graph of a rational function will not intersect any vertical asymptote. It is possible that the graph may intersect a horizontal or slant asymptote. Check by setting the rational function equal to the function for the asymptote and solve. If there is no solution, the graphs do not intersect.

5. Check values of x within each interval to determine the behavior of the function. Complete the graph.

For the following find:

- intercepts
- asymptotes
- additional points as needed

and graph the following:

1. $f(x) = \dfrac{x+1}{x^2+2x-3}$

2. $f(x) = \dfrac{4x^2}{x^2+3}$

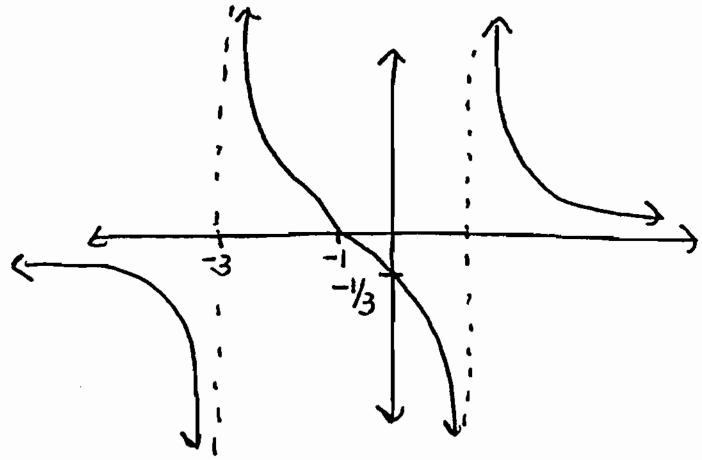
3. $f(x) = \dfrac{x^2+1}{x^2+x-2}$

4. $f(x) = \dfrac{2x^3+5x^2+1}{x^2+x+3}$

Solutions to graphs:

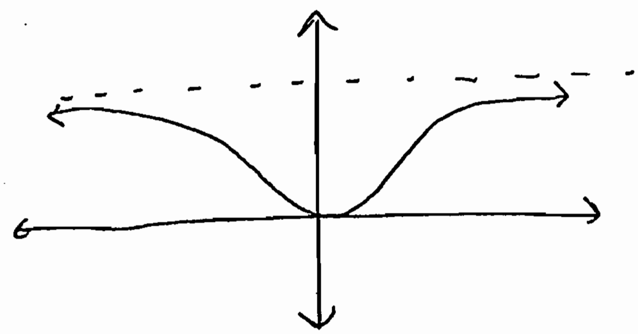
1. $f(x) = \dfrac{x+1}{x^2+2x-3}$

Intercepts: (-1, 0) , $(0, \frac{-1}{3})$
Asymptotes: x = 1, x = -3
y = 0



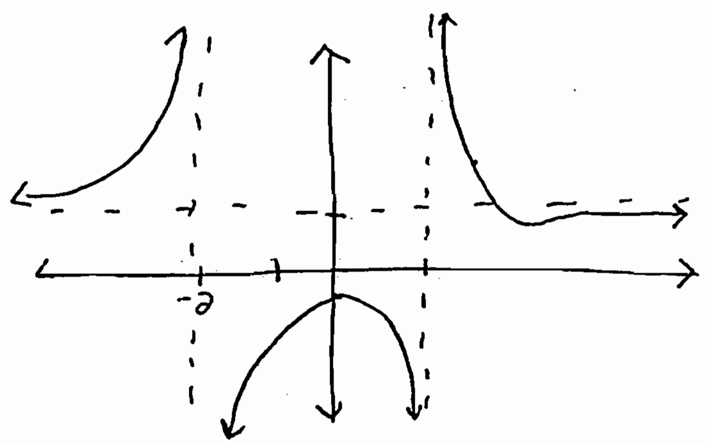
2. $f(x) = \dfrac{4x^2}{x^2+3}$

Intercepts: (0, 0)
Asymptote: y = 4



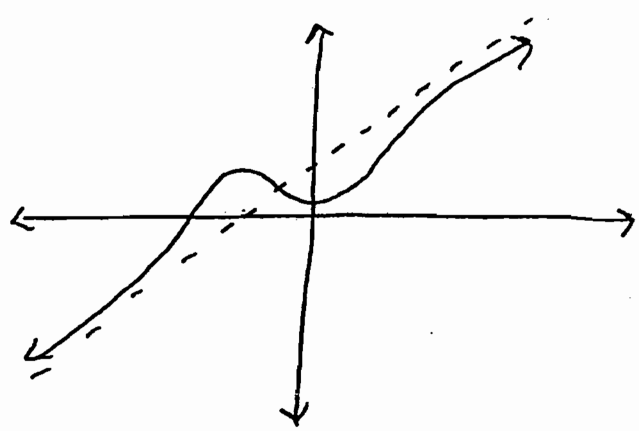
3. $f(x) = \dfrac{x^2+1}{x^2+x-2}$

Intercept: $(0, \frac{-1}{2})$
Asymptotes: x = -2, x = 1, y = 1
Crosses y = 1 at (3,1)



4. $f(x) = \dfrac{2x^3+5x^2+1}{x^2+x+3}$

Intercept: $(0, \frac{1}{3})$
x intercept between
x = -2 and x = -3
Asymptote: y = 2x + 3
crosses at $(\frac{-8}{9}, 1)$



Method 1: Write with a common base and use the one to one property of exponents.

Example: Solve $9^{x-3} = 27^{2x}$

$(3^2)^{x-3} = (3^3)^{2x}$ Get a common base

$3^{2x-6} = 3^{6x}$

$2x - 6 = 6x$ Use one to one property

$-6 = 4x$

$\frac{-3}{2} = x$

Method 2: Take the natural log of both sides and solve.

Example: Solve $3^x\, 2^{x-1} = 5$

$\ln(3^x\, 2^{x-1}) = \ln 5$ Take logs of both sides

$\ln 3^x + \ln 2^{x-1} = \ln 5$

$x \ln 3 + (x-1) \ln 2 = \ln 5$ Simplify

$x \ln 3 + x \ln 2 - \ln 2 = \ln 5$ Distribute

$x (\ln 3 + \ln 2) = \ln 5 + \ln 2$ Solve

$x = \frac{\ln 5 + \ln 2}{\ln 3 + \ln 2}$

$x = \frac{\ln 10}{\ln 6}$

Note: Equations may require algebraic manipulations before they can be solved.

Example: *Solve* $e^{2x} + 2e^x - 3 = 0$

$(e^x + 3)(e^x - 1) = 0$ Factor

$e^x = -3 \qquad e^x = 1$ Solve

$\ln e^x = \ln 1$

x = 0

Since you can't take the log of -3, the only solution is x = 0.

College Algebra/Trig. Review **Exponential Equations**

Solve the following for x.

1.	$8^{2x-3} = 4^{8x}$	1.	$x = -\frac{9}{10}$
2.	$3^x = 7$	2.	$x = 1.77$
3.	$5^{x-2} = 3^x$	3.	$x = 6.30$
4.	$2^{x+4}\ 3^x = 5^x$	4.	$x = -15.21$
5.	$e^{2x} - 3e^x + 2 = 0$	5.	$x = 0, .69$
6.	$\frac{5^x}{3^{x+2}} = 1$	6.	$x = 4.30$
7.	$8e^x + 4 = 7e^x - 2$	7.	No solution
8.	$8^{2x-3} = 4^{3x}$	8.	No solution

Important properties:

A. To change forms

$\log_a b = c \Leftrightarrow b = a^c$ Example: $\log_2 8 = 3 \Leftrightarrow 8 = 2^3$

B. To combine logs

1. $\log_a b + \log_a c = \log_a (bc)$

2. $\log_a b - \log_a c = \log_a \left(\frac{b}{c}\right)$

3. $\log_a b^c = c \log_a b$

Examples: 1.

$$\log_a \sqrt{\frac{xy}{z}} = \log_a \left(\frac{xy}{z}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2} \log_a \left(\frac{xy}{z}\right)$$

$$= \frac{1}{2} \left[\log_a (xy) - \log_a z\right]$$

$$= \frac{1}{2} \left[\log_a x + \log_a y - \log_a z\right]$$

2.

$$3 \log_a x + 2 \log_a y - 5 \log_a z = \log_a x^3 + \log_a y^2 - \log_a z^5$$

$$= \log_a \frac{x^3 y^2}{z^5}$$

C. Inverse Properties:

1. $\log_b b^c = c$

2. $b^{\log_b c} = c$

1. Change $\log_3 y = x$ to exponential form.

1. $3^x = y$

2. Change $x^{3+y} = 2$ to logarithmic form.

2. $\log_x 2 = 3 + y$

3. Write the following as a single log.

$$\frac{1}{2}\log_3 (x+y) - \log_3 x + 2\log_3 y$$

3. $\log_3 \dfrac{y^2\sqrt{x+y}}{x}$

4. Expand the following.

$$\log_2 \frac{x-y}{x^3 y^2}$$

4. $\log_2 (x-y) - 3\log_2 x - 2\log_2 y$

5. *If* $\log_a 2 = B$ *and* $\log_a 3 = C$ *find:*

a. $\log_a 8$

b. $\log_a 36$

c. $\log_a \dfrac{4}{9}$

d. $\log_a \sqrt{6}$

5. a. 3B

b. 2B + 2C

c. $\dfrac{B^2}{C^2}$

d. $\dfrac{1}{2}(B + C)$

Logarithmic equations are solved by changing to exponential form and then solving the resulting equation.

Example: Solve $\log_8 (x - 5) = 2$

$x - 5 = 8^2$	Change to exponential form
$x - 5 = 64$	Solve the resulting equation
$x = 69$	

When equations involve more than one logarithm, the logs must be combined first.

Examples: 1. Solve $\log_2 (x + 2) - \log_2 (x - 4) = 2$

$\log_2 \frac{x+2}{x-4} = 2$	Combine logs
$\frac{x+2}{x-4} = 2^2$	Change to exponential form
$\frac{x+2}{x-4} = 4$	Solve the resulting equation

$x + 2 = 4x - 16$

$18 = 3x$

$6 = x$

3. Solve $\log_5 x + \log_5 (x + 4) = 1$

$\log_5 x(x + 4) = 1$

$x(x + 4) = 5$

$x^2 + 4x - 5 = 0$

$(x + 5)(x - 1) = 0$

$x = -5, 1$

Note: You can't take the log of a negative solution and so the only solution is $x = 1$.

College Algebra/Trig. Review **Logarithmic Equations**

Solve the following equations.

1. $\log_2 x = 5$ — 1. $x = 32$

2. $\log_4 8 = x$ — 2. $x = \frac{3}{2}$

3. $\log_x 64 = 3$ — 3. $x = 4$

4. $\log_2 x + \log_2 (x + 3) = 2$ — 4. $x = 1$

5. $\log_3 (x + 2) - \log_3 (x - 4) = 2$ — 5. $x = \frac{19}{4}$

6. $\log_2 (x + 4) - \log_2 (x + 1) = 3$ — 6. no solution

7. $\log_7 (x + 2) + \log_7 (x + 4) = \log_7 (x^2 + 20)$ — 7. $x = 2$

Part 2

TRIGONOMETRY

- Angles may be measured in degrees or radians. When units are omitted the measurements are in radians.

- Conversion factors are:

radians to	degrees to
degrees: $\frac{180^0}{\pi}$	radians: $\frac{\pi}{180^0}$

Examples: 1. $75^o \bullet \frac{\pi}{180^0} = \frac{5\pi}{12}$

2. $\frac{4\pi}{9} \bullet \frac{180^0}{\pi} = 80^o$

- The following conversions should be memorized:

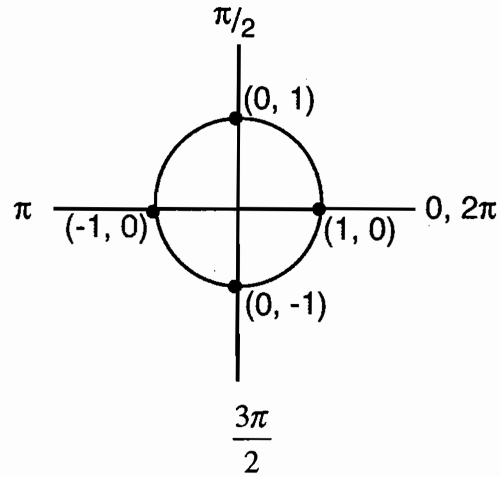
$0 = 0^o$	$\frac{\pi}{6} = 30^o$
$\frac{\pi}{4} = 45^o$	$\frac{\pi}{3} = 60^o$
$\frac{\pi}{2} = 90^o$	$\pi = 180^o$
$\frac{3\pi}{2} = 270^o$	$2\pi = 360^o$

- If sin θ and cos θ are known, then all other functions can be found using the identities.

- If (x, y) is a point on the terminal side of an angle θ and (x,y) is on the unit circle (center (0,0) and radius = 1), then

$$\sin\theta = y \quad \cos\theta = x$$

- Trig values of quadrantal angles (angles whose terminating side lies on an axis) can be computed using the following points on the unit circle:

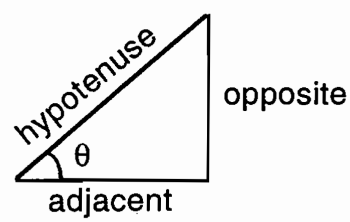


Examples: $\sin \frac{\pi}{2} = 1$

$\cos \frac{3\pi}{2} = 0$

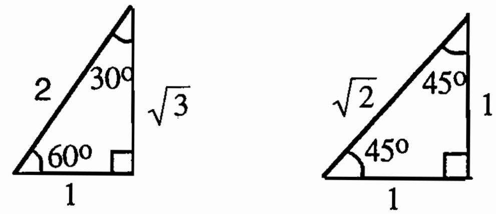
$\tan \frac{3\pi}{2} = \frac{\sin \frac{3\pi}{2}}{\cos \frac{3\pi}{2}} = \frac{-1}{0}$ = undefined

- In the right triangle labeled below:



$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

- Special triangles are:



- Using the special triangles, we obtain:

$$\cos 30^\circ = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\sin 30^\circ = \sin \frac{\pi}{6} = \frac{1}{2}$$

$$\cos 60^\circ = \cos \frac{\pi}{3} = \frac{1}{2}$$

$$\sin 60^\circ = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos 45^\circ = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\sin 45^\circ = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

These values should be memorized or derived very quickly.

- A reference angle is the acute angle found by drawing a perpendicular line to the x axis.
- The trig value of an angle and its reference angle are the same except possibly the sign.
- The sign of the answer can be determined by the quadrant θ is in. The positive trig values in each quadrant are as follows

Q II $\sin \theta > 0$ $\csc \theta > 0$	Q I all > 0
$\tan \theta > 0$ $\cot \theta > 0$ Q III	$\cos \theta > 0$ $\sec \theta > 0$ Q IV

Example: 1. $\sin\left(\frac{5\pi}{4}\right) = \pm \sin\left(\frac{\pi}{4}\right)$ reference angle = $\frac{\pi}{4}$

$= \frac{-\sqrt{2}}{2}$ $\frac{5\pi}{4}$ *is in QIII*

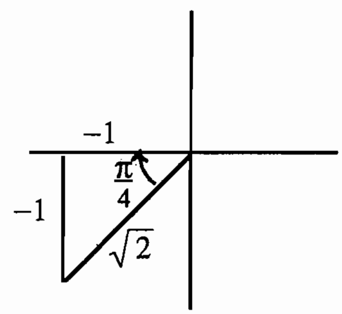
2. $\cos\left(\frac{11\pi}{6}\right) = \pm \cos\left(\frac{\pi}{6}\right)$ reference angle = $\frac{\pi}{6}$

$= \frac{\sqrt{3}}{2}$ $\frac{11\pi}{6}$ *is in Q IV*

- Alternate triangle approach

  Values can also be determined by sketching triangles with θ being the reference angle.

Example: $\sin \frac{5\pi}{4}$



$\sin \frac{5\pi}{4} = \frac{-1}{\sqrt{2}}$

$= \frac{-\sqrt{2}}{2}$

College Algebra/Trig. Review **Degrees/Radians**

Change the following degrees to radians.

1. $\theta = 85^o$
2. $\theta = -225^o$
3. $\theta = 410^o$
4. $\theta = 70^o$
5. $\theta = -28^o$

1. $\theta = \dfrac{17\pi}{36}$
2. $\theta = \dfrac{-5\pi}{4}$
3. $\theta = \dfrac{41\pi}{18}$
4. $\theta = \dfrac{7\pi}{18}$
5. $\theta = \dfrac{-7\pi}{45}$

Change the following radians to degrees.

1. $\theta = \dfrac{8\pi}{3}$
2. $\theta = \dfrac{6\pi}{5}$
3. $\theta = \dfrac{-5\pi}{9}$
4. $\theta = \dfrac{7\pi}{8}$
5. $\theta = \dfrac{-5\pi}{12}$

1. $\theta = 480^o$
2. $\theta = 216^o$
3. $\theta = -100^o$
4. $\theta = \dfrac{315^o}{2}$ or $\theta = 157.5^o$
5. $\theta = -75^o$

College Algebra/Trig. Review

1. $\sin(5\pi)$
2. $\cos\left(\frac{-7\pi}{2}\right)$
3. $\tan(3\pi)$
4. $\cot(-7\pi)$
5. $\sec(11\pi)$
6. $\csc\left(\frac{9\pi}{2}\right)$
7. $\tan\left(\frac{7\pi}{2}\right)$
8. $\csc(-15\pi)$
9. $\cot\left(\frac{5\pi}{2}\right)$
10. $\sec\left(\frac{-7\pi}{2}\right)$

Evaluating Trig. Values

1. 0
2. 0
3. 0
4. undefined
5. -1
6. 1
7. undefined
8. undefined
9. 0
10. undefined

Evaluate the following:

1. $\sin\left(\frac{2\pi}{3}\right)$
2. $\cos\left(\frac{5\pi}{4}\right)$
3. $\sec\left(\frac{\pi}{6}\right)$
4. $\csc\left(\frac{-\pi}{6}\right)$
5. $\tan\left(\frac{11\pi}{4}\right)$
6. $\cot\left(\frac{-\pi}{3}\right)$
7. $\sin\left(\frac{9\pi}{4}\right)$
8. $\cos\left(\frac{-8\pi}{3}\right)$
9. $\tan\left(\frac{5\pi}{6}\right)$
10. $\sec\left(\frac{-3\pi}{4}\right)$

Answers:

1. $\frac{\sqrt{3}}{2}$
2. $\frac{-\sqrt{2}}{2}$
3. $\frac{2}{\sqrt{3}}$
4. - 2
5. 1
6. $\frac{-1}{\sqrt{3}}$
7. $\frac{\sqrt{2}}{2}$
8. $\frac{-1}{2}$
9. $-\frac{1}{\sqrt{3}}$
10. $-\sqrt{2}$

Definitions:

1. $y = \sin^{-1}x \iff \sin y = x$

   $-1 \leq x \leq 1$

   $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

   Examples: $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ since $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

   $\sin^{-1}\left(\frac{-\sqrt{2}}{2}\right) = \frac{-\pi}{4}$ since $\sin\left(\frac{-\pi}{4}\right) = \frac{-\sqrt{2}}{2}$

2. $y = \tan^{-1} x \iff \tan y = x$

   x is any real number

   $\frac{-\pi}{2} < y < \frac{\pi}{2}$

   Examples: $\tan^{-1}(-1) = -\frac{\pi}{4}$ since $\tan\left(\frac{-\pi}{4}\right) = -1$

   $\tan^{-1}(\sqrt{3}) = -\frac{\pi}{3}$ since $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

3. $y = \cos^{-1} x \iff \cos y = x$

   $-1 \leq x \leq 1$

   $0 \leq y \leq \pi$

   Examples: $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$ since $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

   $\cos^{-1}\left(\frac{-\sqrt{2}}{2}\right) = \frac{3\pi}{4}$ since $\cos\left(\frac{3\pi}{4}\right) = \frac{-\sqrt{2}}{2}$

Evaluate the following:

1.	$\sin^{-1}\left(\frac{1}{2}\right)$	1.	$\theta = \frac{\pi}{6}$
2.	$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$	2.	$\theta = \frac{\pi}{4}$
3.	$\tan^{-1}(1)$	3.	$\theta = \frac{\pi}{4}$
4.	$\cos^{-1}\left(\frac{-1}{2}\right)$	4.	$\theta = \frac{2\pi}{3}$
5.	$\sin^{-1}(0)$	5.	$\theta = 0$
6.	$\sin^{-1}\left(\frac{-3}{2}\right)$	6.	undefined
7.	$\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$	7.	$\theta = \frac{-\pi}{3}$
8.	$\tan^{-1}\left(-\sqrt{3}\right)$	8.	$\theta = \frac{-\pi}{3}$
9.	$\cos^{-1}(1)$	9.	$\theta = 0$
10.	$\cos^{-1}\left(\frac{5}{3}\right)$	10.	undefined

The following identities should be memorized.

1. Quotient Identities:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

2. Reciprocal Identities:

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\sec\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

3. Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

4. Sum and Difference Identities:

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

5. Double Angle Identities:

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$= 1 - 2\sin^2\theta$$

$$= 2\cos^2\theta - 1$$

6. Power Reducing Formulas:

$$\cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta))$$

$$\sin^2 \theta = \frac{1}{2}(1 - \cos(2\theta))$$

7. Half Angle Identities:

$$\cos \theta = \pm \sqrt{\frac{1 + \cos(2\theta)}{2}}$$

$$\sin \theta = \pm \sqrt{\frac{1 - \cos(2\theta)}{2}}$$

Identities can be used to find exact values of many trig. functions.

Examples: 1. If $\sin\theta = \frac{5}{8}$ and $\tan\theta < 0$ find $\cos\theta$

$\sin\theta = \frac{5}{8}$, $\tan\theta < 0 \Rightarrow \theta$ is in quadrant 2 and $\cos\theta$ is negative

$$\sin^2\theta + \cos^2\theta = \Rightarrow \frac{25}{64} + \cos^2\theta = 1$$

$$\cos^2\theta = \frac{39}{64}$$

$$\cos\theta = \frac{-\sqrt{39}}{8}$$

2. If $\cos\theta = \frac{1}{3}$ and $\sin\theta = \frac{2\sqrt{2}}{3}$ find $\tan\theta$ and $\sec\theta$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \tan\theta = \frac{\frac{2\sqrt{2}}{3}}{\frac{1}{3}}$$

$$= 2\sqrt{2}$$

$$\sec\theta = \frac{1}{\cos\theta} \Rightarrow \sec\theta = \frac{1}{\frac{1}{3}}$$

$$= 3$$

3. Evaluate: $\sin(195^\circ)$

$$\sin(195^\circ) = \sin(150^\circ + 45^\circ)$$

$$= \sin(150^\circ)\cos(45^\circ) + \cos(150^\circ)\sin(45^\circ)$$

$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \left(\frac{-\sqrt{3}}{2}\right)\cdot\frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2}-\sqrt{6}}{4}$$

4. Evaluate: $1 - 2\sin^2\left(\frac{-\pi}{8}\right)$

$$1 - 2\sin^2\left(\frac{-\pi}{8}\right) = \cos\ 2\left(\frac{-\pi}{8}\right)$$

$$= \cos\left(\frac{-\pi}{4}\right)$$

$$= \frac{\sqrt{2}}{2}$$

Use the identities to solve the following:

1. If sin $\theta = \frac{1}{3}$ *and* $\tan\theta < 0$
find $\cos\theta, \csc\theta$ *and* $\cot\theta$

2. Find the exact value of
$\cos\left(\frac{\pi}{12}\right)$ *Note*: $\frac{\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}$

3. If sin $\theta = \frac{3}{5}, \frac{\pi}{2} < \theta < \pi$
Find the exact value of
sin (2θ) *and* $\cos(2\theta)$

4. Evaluate cos $\left(\frac{\alpha}{2}\right)$ *if*
$\cos\alpha = \frac{-3}{5}$ *and* $\pi < \alpha < \frac{3\pi}{2}$

5. Find the exact value of
$\sin\frac{\pi}{9}\cos\frac{\pi}{18} + \cos\frac{\pi}{9}\sin\frac{\pi}{18}$

Answers:

1. $\cos\theta = \frac{-2\sqrt{2}}{3}, \csc\theta = 3,$
$\cot\theta = -2\sqrt{2}$

2. $\frac{1}{4}\left(\sqrt{6} + \sqrt{2}\right)$

3. $\sin(2\theta) = \frac{-24}{25}, \cos(2\theta) = \frac{7}{25}$

4. $\cos\left(\frac{\alpha}{2}\right) = \frac{-\sqrt{5}}{5}$

5. $\frac{1}{2}$

To prove identities:

- Work on one side at a time
- Use basic identities

  Example: $\sin^2 x + \cos^2 x$ can be replaced with 1
- Use algebraic manipulations

  Example: Get common denominators
  Clear complex fractions
  Rationalize by multiplying by the conjugate
  Factor
  Multiply factors
- Change to sines and cosines

Examples: 1. Prove $\tan^2 x \bullet \cos^2 x - 1 = -\cos^2 x$

Pf: $\tan^2 x \bullet \cos^2 x - 1 = \dfrac{\sin^2 x}{\cos^2 x} \bullet \cos^2 x - 1$

$= \sin^2 x - 1$

$= -\cos^2 x$

2. Prove $\dfrac{1}{1-\sin x} = \sec x(\sec x + \tan x)$

Pf: $\dfrac{1}{1-\sin x} = \dfrac{1}{1-\sin x} \bullet \dfrac{1+\sin x}{1+\sin x}$

$= \dfrac{1+\sin x}{1-\sin^2 x}$

$= \dfrac{1+\sin x}{\cos^2 x}$

$= \dfrac{1}{\cos^2 x} + \dfrac{\sin x}{\cos^2 x}$

$= \sec^2 x + \dfrac{\sin x}{\cos x} \bullet \dfrac{1}{\cos x}$

$= \sec^2 x + \tan x \sec x$

$= \sec x(\sec x + \tan x)$

Prove the following identities:

1. $$\frac{\sin\theta\csc\theta}{\cos\theta\tan\theta}=\csc\theta$$

2. $$\frac{1+\tan^2\theta}{1+\cot^2\theta}=\tan^2\theta$$

3. $$\frac{\cot\theta}{1-\cot\theta}=\frac{1}{\tan\theta-1}$$

4. $$\cos\theta-\sec\theta=-\sin\theta\tan\theta$$

5. $$\frac{1}{1-\cos\theta}=\csc\theta\,(\csc\theta+\cot\theta)$$

6. $$\sec\theta+\tan\theta=\frac{\cos\theta}{1-\sin\theta}$$

7. $$2\sin^2\theta+\cos^2\theta-1=\sin^2\theta$$

8. $$\frac{\sin^2 x-\cos^2 x}{\sin^2 x-\sin x\cos x}=1+\cot x$$

1.

$$\frac{\sin\theta\csc\theta}{\cos\theta\tan\theta} = \frac{\sin\theta \bullet \dfrac{1}{\sin\theta}}{\cos\theta \bullet \dfrac{\sin\theta}{\cos\theta}}$$

$$= \frac{1}{\sin\theta}$$

$$= \csc\theta$$

2.

$$\frac{1+\tan^2\theta}{1+\cot^2\theta} = \frac{\sec^2\theta}{\csc^2\theta}$$

$$= \frac{\dfrac{1}{\cos^2\theta}}{\dfrac{1}{\sin^2\theta}}$$

$$= \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \tan^2\theta$$

3.

$$\frac{\cot\theta}{1-\cot\theta} = \frac{\dfrac{1}{\tan\theta}}{1-\dfrac{1}{\tan\theta}}$$

$$= \frac{1}{\tan\theta - 1}$$

4.

$$\cos\theta - \sec\theta = \cos\theta - \frac{1}{\cos\theta}$$

$$= \frac{\cos^2\theta - 1}{\cos\theta}$$

$$= \frac{-\sin^2\theta}{\cos\theta}$$

$$= \frac{-\sin\theta}{\cos\theta} \bullet \sin\theta$$

$$= -\sin\theta\tan\theta$$

5.

$$\frac{1}{1-\cos\theta} = \frac{1}{1-\cos\theta} \bullet \frac{1+\cos\theta}{1+\cos\theta}$$

$$= \frac{1+\cos\theta}{1-\cos^2\theta}$$

$$= \frac{1+\cos\theta}{\sin^2\theta}$$

$$= \frac{1}{\sin^2\theta} + \frac{\cos\theta}{\sin\theta} \bullet \frac{1}{\sin\theta}$$

$$= \csc^2\theta + \cot\theta\csc\theta$$

$$= \csc\theta\,(\csc\theta + \cot\theta)$$

6. $$\sec\theta + \tan\theta = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

$$= \frac{(1+\sin\theta)(1-\sin\theta)}{\cos\theta\,(1-\sin\theta)}$$

$$= \frac{(1-\sin^2\theta)}{\cos\theta\,(1-\sin\theta)}$$

$$= \frac{\cos^2\theta}{\cos\theta\,(1-\sin\theta)}$$

$$= \frac{\cos\theta}{1-\sin\theta}$$

7. $$2\sin^2\theta + \cos^2\theta - 1 = 2\sin^2\theta + 1 - \sin^2\theta - 1$$

$$= \sin^2\theta$$

8. $$\frac{\sin^2 x - \cos^2 x}{\sin^2 x - \sin x \cos x} = \frac{(\sin x + \cos x)(\sin x - \cos x)}{\sin x(\sin x - \cos x)}$$

$$= \frac{\sin x + \cos x}{\sin x}$$

$$= \frac{\sin x}{\sin x} + \frac{\cos x}{\sin x}$$

$$= 1 + \cot x$$

Note: The previous solutions are only one possible solution. Other solutions are possible.

To solve trig equations a variety of algebraic and trigonometric properties are used.

Examples:

1. $\sin^2\theta + \sin\theta - 2 = 0$

   $(\sin\theta + 2)(\sin\theta - 1) = 0$ *Factor*

   $\sin\theta + 2 = 0 \qquad \sin\theta - 1 = 0$ *Solve*

   $\sin\theta = -2 \qquad \sin\theta = 1$

   no solution $\qquad \theta = \pi/2$

2. $\sin^2\theta + \cos\theta = 1$

   $1 - \cos^2\theta + \cos\theta = 1$ *Use trig identity*

   $\cos\theta - \cos^2\theta = 0$

   $\cos\theta\,(1 - \cos\theta) = 0$ *Factor*

   $\cos\theta = 0 \qquad 1 - \cos\theta = 0$ *Solve*

   $\theta = \pi/2,\ 3\pi/2 \qquad \theta = 0$

3. $4\cos^2(3\theta) = 1$ $\quad$ *3θ indicates that you must find solutions in $0 \le \theta < 6\pi$ and the divide by 3.*

   $\cos^2(3\theta) = \frac{1}{4}$

   $\cos(3\theta) = \pm\frac{1}{2}$

$$3\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{14\pi}{3}, \frac{16\pi}{3}, \frac{17\pi}{3}$$

$$\theta = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{14\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9}$$

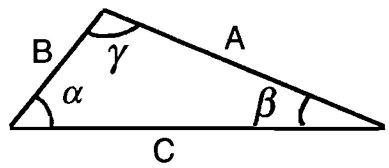
NOTE: If at any time you multiply by a variable or square both sides, answers must be checked for extraneous solutions.

College Algebra/Trig. Review **Trig. Equations**

Solve the following equations for θ where $0 \leq \theta < 2\pi$.

1. $\sin\theta\ (2\cos\theta + 1) = 0$	1. $\theta = 0,\ \pi,\ \frac{2\pi}{3},\ \frac{4\pi}{3}$
2. $\sin^2\theta + \sin\theta = 0$	2. $\theta = 0,\ \pi,\ \frac{3\pi}{2}$
3. $2\cos^2\theta - 3\cos\theta + 1 = 0$	3. $\theta = 0,\ \frac{\pi}{3},\ \frac{5\pi}{3}$
4. $\cos\theta + 2\sin^2\theta = 2$	4. $\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{5\pi}{3}$
5. $2\sin^2(3\theta) = 1$	5. $\theta = \frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{3\pi}{4}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{5\pi}{4}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{7\pi}{4}, \frac{23\pi}{12}$
6. $\cos\theta - \sin\theta = 1$	6. $\theta = 0,\ \frac{3\pi}{2}$
7. $\cos(2\theta) + \sin\theta = 0$	7. $\theta = \frac{7\pi}{6},\ \frac{11\pi}{6},\ \frac{\pi}{2}$
8. $\tan\theta + \cot\theta = 2$	8. $\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

In the following triangle:



$$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$$

Case 1: ASA (angle, side, angle), or SAA (side, angle, angle)
Only one triangle exists.

Example: $\alpha = 40^{o}$, $\beta = 60^{o}$, a = 4

Since $\alpha + \beta + \gamma = 180^{o}$, $\gamma = 80^{o}$

$$\frac{\sin 40^\circ}{4} = \frac{\sin 60^\circ}{b} \quad \textit{and} \quad \frac{\sin 40^\circ}{4} = \frac{\sin 80^\circ}{c}$$

$$b = \frac{4\sin 60^\circ}{\sin 40^\circ} = 5.39 \qquad c = \frac{4\sin 80^\circ}{\sin 40^\circ} = 6.13$$

Case 2: SSA (side, side, angle)
This is the ambiguous case. You may have no triangle, one triangle, or two triangles.

Example 1: a = 2, c = 1, $\gamma = 50^{o}$

$$\frac{\sin\alpha}{2} = \frac{\sin 50^\circ}{1}$$

$\sin\alpha = 2\sin 50^{o}$

$\sin\alpha = 1.53$ (This can't happen, so we have no triangle.)

Example 2. a = 6 b = 8 $\alpha = 35^\circ$

$$\frac{\sin 35^\circ}{6} = \frac{\sin\beta}{8}$$

$$\frac{\sin\beta}{6} = \frac{8\sin 35^\circ}{6}$$

$\beta_1 = 49.9^\circ$ $\beta_2 = 180^\circ - 49.9^\circ$

(QI answer) $= 130.1^\circ$ (QII answer)

Since in both cases we have $\alpha + \beta < 180^\circ$, they can both be used and we have two triangles.

Example 3. a = 3 b = 2 $\alpha = 40^\circ$

$$\frac{\sin 40^\circ}{3} = \frac{\sin\beta}{2}$$

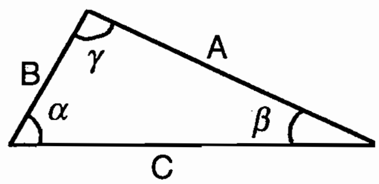
$$\sin\beta = \frac{2\sin 40^\circ}{3}$$

$\beta_1 = 25.4^\circ$ $\beta_2 = 180^\circ - 25.4^\circ$

$= 154.6^\circ$

Since $\alpha + \beta_2 = 40^\circ + 154.6^\circ = 194.6^\circ$, we can't have the 2nd triangle.

In the following triangle:



$A^2 = B^2 + C^2 - 2BC \cos \alpha$

$B^2 = C^2 + A^2 - 2\,AC \cos \beta$

$C^2 = A^2 + B^2 - 2\,AB \cos \gamma$

Case 1: SAS (side, angle, side)

Example: $a = 2 \quad b = 3 \quad \gamma = 60^\circ$

$C^2 = 4 + 9 - 2 \bullet 2 \bullet 3 \cos 60^\circ$

$= 7$

$C = \sqrt{7}$

* To find other angles use law of cosines not sines to avoid the ambiguous case.

Case 2: SSS (side, side, side)

Example: $a = 4,\ b = 3,\ c = 6$

$$\cos \alpha = \frac{b^2 + c^2 - a^2}{abc}$$

$$= \frac{29}{36}$$

$$\alpha = \cos^{-1}\left(\frac{29}{36}\right)$$

$$= 36.3^\circ$$

* The 2nd angle can be found the same way. Then use $\alpha + \beta + \gamma = 180^\circ$ to get the 3rd angle.

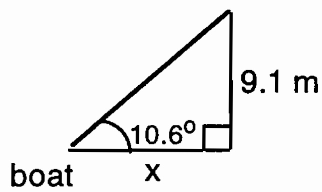
College Algebra/Trig. Review **Law of Sines and Cosines**

Solve the following triangles:

1. $c = 60$, $A = 10^o$, $C = 135^o$	1. $B = 35^o$, $a = 14.73$, $b = 48.67$
2. $A = 40^o$, $B = 60^o$, $a = 4$	2. $C = 80^o$, $b = 5.39$, $c = 6.13$
3. $A = 58^o$, $a = 11.4$, $b = 12.8$	3. Triangle 1: $B = 72.21^o$, $C = 49.79^o$, $c = 10.27$ Triangle 2: $B = 107.79^o$, $C = 14.21^o$, $c = 3.30$
4. $A = 58^o$, $a = 42.4$, $b = 50$	4. No triangle
5. $A = 38^o$, $a = 9$, $b = 7$	5. $B = 28.61^o$, $C = 113.39^o$, $c = 13.42$
6. $a = 55$, $b = 25$, $c = 72$	6. $C = 123.91^o$, $A = 39.35^o$, $B = 16.74^o$

1. Using right triangle trig.

Example: The angle of elevation from a boat to the top of a lighthouse is 10.6°. The lighthouse is 9.1 m in height. Find the distance from the base of the lighthouse to the boat.



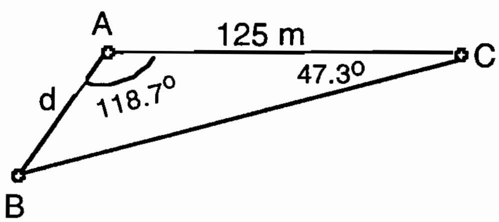
$$\tan 10.6^\circ = \frac{9.1}{x}$$

$$x = \frac{9.1}{\tan 10.6^\circ}$$

$$x = 48.6 \text{ m}$$

2. Using law of sines

Example: Two points A and B are on opposite sides of a swamp. In order to find the distance AB, a point C is located on the same side of the swamp as A and 125 meters from A. Angle C is determined to be 47.3° and Angle A is determined to be 118.7°. Find the length AB across the swamp.



$$\angle B = 180 - (118.7 + 47.3)$$

$$= 14^\circ$$

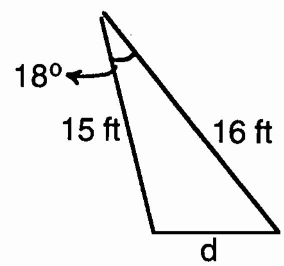
$$\frac{\sin 14^\circ}{125} = \frac{\sin 47.3^\circ}{d}$$

$$d = \frac{125 \sin 47.3^\circ}{\sin 14^\circ}$$

$$d = 379.7$$

3. Using law of cosines:

Example: A pole vaulter leans his 16 ft. pole against a sloping wall making an angle of 18° with the wall. If the point at which the pole touches the wall is 15 ft. up from where the wall meets the ground, how far away from the wall is the bottom of the pole?



$d^2 = (15)^2 + (16)^2 - 2\,(15)\,(16)\cos 18^\circ$

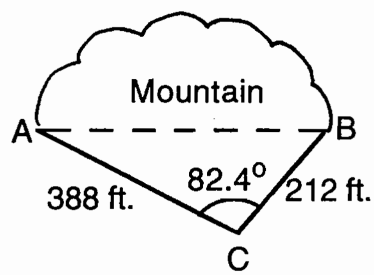
$d = 4.9$ ft.

Solve the following:

1. A tree casts a shadow 532 ft. long. Find the height of the tree if the angle of elevation of the sun is 25.7°.

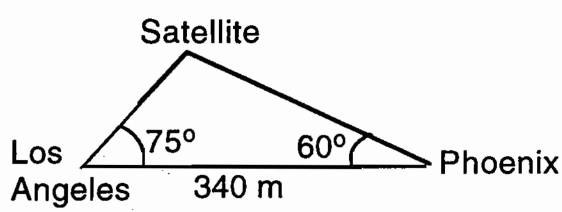
   Answer: 256 feet

2. A tunnel is to be built through a mountain. To estimate the length of the tunnel, a surveyor makes the measurements shown in the figure below. Find the length of the tunnel.



   Answer: 416.8 feet

3. A satellite orbiting the earth passes directly over Phoenix and Los Angeles which is 340 mi. apart. When the satellite is between the two stations, the angles of elevation are 60° at Phoenix and 75° at Los Angeles (see figure). How far is the satellite from Los Angeles?



   Answer: 416.4 miles